Biophysics: Physics for Life Sciences

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Solutions for Home work Problems

Fluid Mechanics: Chapter 11

atmospheric pressure plus that due to the water. The water pressure P_2 at the depth h is related to the pressure P_1 at the surface by Equation 11.4, $P_2 = P_1 + \rho gh$, where ρ is the density of the fluid, g is the magnitude of the acceleration due to gravity, and h is the depth. This relation can be used directly to find the depth.

SOLUTION We are given that the maximum difference in pressure between the outside and inside of the lungs is one-twentieth of an atmosphere, or $P_2 - P_1 = \frac{1}{20}\left(1.01\times10^5 \text{ Pa}\right)$. Solving Equation 11.4 for the depth h gives

$$h = \frac{P_2 - P_1}{\rho g} = \frac{\frac{1}{20}\left(1.01\times10^5 \text{ Pa}\right)}{\left(1025 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)} = \boxed{0.50 \text{ m}}$$

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26. ***REASONING*** The pressure P_2 at a lower point in a static fluid is related to the pressure P_1 at a higher point by Equation 11.4, $P_2 = P_1 + \rho gh$, where ρ is the density of the fluid, g is the magnitude of the acceleration due to gravity, and h is the difference in heights between the two points. This relation can be used directly to find the pressure in the artery in the brain.

SOLUTION Solving Equation 11.4 for pressure P_1 in the brain (the higher point), gives

$$P_1 = P_2 - \rho gh = 1.6\times10^4 \text{ Pa} - \left(1060 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(0.45 \text{ m}\right) = \boxed{1.1\times10^4 \text{ Pa}}$$

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27. SSM ***REASONING*** The pressure P at a distance h beneath the water surface at the vented top of the water tower is $P = P_{\text{atm}} + \rho gh$ (Equation 11.4). We note that the value for h in this expression is different for the two houses and must take into account the diameter of the spherical reservoir in each case.

SOLUTION

a. The pressure at the level of house A is given by Equation 11.4 as $P_A = P_{\text{atm}} + \rho gh_A$. Now the height h_A consists of the 15.0 m plus the diameter d of the tank. We first calculate the radius of the tank, from which we can infer d. Since the tank is spherical, its full mass is given by $M = \rho V = \rho[(4/3)\pi r^3]$. Therefore,

$$r^3 = \frac{3M}{4\pi\rho} \quad \text{or} \quad r = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left[\frac{3(5.25\times10^5 \text{kg})}{4\pi(1.000\times10^3 \text{kg/m}^3)}\right]^{1/3} = 5.00 \text{ m}$$

Therefore, the diameter of the tank is 10.0 m, and the height h_A is given by

$$h_A = 10.0 \text{ m} + 15.0 \text{ m} = 25.0 \text{ m}$$

According to Equation 11.4, the gauge pressure in house A is, therefore,

$$P_A - P_{atm} = \rho g h_A = (1.000\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(25.0 \text{ m}) = \boxed{2.45\times10^5 \text{ Pa}}$$

b. The pressure at house B is $P_B = P_{atm} + \rho g h_B$, where

$$h_B = 15.0 \text{ m} + 10.0 \text{ m} - 7.30 \text{ m} = 17.7 \text{ m}$$

According to Equation 11.4, the gauge pressure in house B is

$$P_B - P_{atm} = \rho g h_B = (1.000\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(17.7 \text{ m}) = \boxed{1.73\times10^5 \text{ Pa}}$$

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28. ***REASONING*** If the pressure is P_1 at a certain point in a static fluid (density $= \rho$), the pressure P_2 in the fluid at a distance h beneath this point is $P_2 = P_1 + \rho g h$ (Equation 11.4). As discussed in Section 11.4, this expression becomes $P_{atm} = \rho_{mercury} g h_{mercury}$ when applied to the mercury barometer shown in Figure 11.11 of the text.

SOLUTION Using Equation 11.4 with $P_2 = P_{ground}$ and $P_1 = P_{roof}$, we have

$$P_{ground} = P_{roof} + \rho_{air} g h_{air} \quad \text{or} \quad P_{ground} - P_{roof} = \rho_{air} g h_{air}$$

In this result, h_{air} is the height of a column of air that equals the height of the building. We can also express $P_{ground} - P_{roof}$ using the mercury barometer, which indicates that $P_{ground} - P_{roof}$ corresponds to a column of mercury that has a height of $h_{mercury} = 760.0 - 747.0 = 13.0 \text{ mm}$. Thus, we have

$$P_{ground} - P_{roof} = \rho_{mercury} g h_{mercury}$$

Equating the two expressions for $P_{ground} - P_{roof}$, we obtain

$$P_{ground} - P_{roof} = \rho_{air} g h_{air} = \rho_{mercury} g h_{mercury} \quad \text{or} \quad \rho_{air} h_{air} = \rho_{mercury} h_{mercury}$$

Taking the density of mercury from Table 11.1 in the text and solving for h_{air} gives

$$h_{air} = \frac{\rho_{mercury} h_{mercury}}{\rho_{air}} = \frac{\left(13\,600 \text{ kg/m}^3\right)\left(13.0 \times 10^{-3} \text{ m}\right)}{1.29 \text{ kg/m}^3} = \boxed{137 \text{ m}}$$

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44. ***REASONING*** The paperweight weighs less in water than in air, because of the buoyant force F_B of the water. The buoyant force points upward, while the weight points downward, leading to an effective weight in water of $W_{\text{In water}} = W - F_B$. There is also a buoyant force when the paperweight is weighed in air, but it is negligibly small. Thus, from the given weights, we can obtain the buoyant force, which is the weight of the displaced water, according to Archimedes' principle. From the weight of the displaced water and the density of water, we can obtain the volume of the water, which is also the volume of the completely immersed paperweight.

SOLUTION We have

$$W_{\text{In water}} = W - F_B \quad \text{or} \quad F_B = W - W_{\text{In water}}$$

According to Archimedes' principle, the buoyant force is the weight of the displaced water, which is mg, where m is the mass of the displaced water. Using Equation 11.1, we can write the mass as the density times the volume or $m = \rho V$. Thus, for the buoyant force, we have

$$F_B = W - W_{\text{In water}} = \rho V g$$

Solving for the volume and using $\rho = 1.00 \times 10^3$ kg/m^3 for the density of water (see Table 11.1), we find

$$V = \frac{W - W_{\text{In water}}}{\rho g} = \frac{6.9\ \text{N} - 4.3\ \text{N}}{\left(1.00 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{2.7 \times 10^{-4}\ \text{m}^3}$$

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45. **SSM** ***REASONING*** According to Equation 11.1, the density of the life jacket is its mass divided by its volume. The volume is given. To obtain the mass, we note that the person wearing the life jacket is floating, so that the upward-acting buoyant force balances the downward-acting weight of the person and life jacket. The magnitude of the buoyant force is the weight of the displaced water, according to Archimedes' principle. We can express each of the weights as mg (Equation 4.5) and then relate the mass of the displaced water to the density of water and the displaced volume by using Equation 11.1.

SOLUTION According to Equation 11.1, the density of the life jacket is

$$\rho_J = \frac{m_J}{V_J} \tag{1}$$

Since the person wearing the life jacket is floating, the upward-acting buoyant force F_B balances the downward-acting weight W_P of the person and the weight W_J of the life jacket. The buoyant force has a magnitude that equals the weight W_{H_2O} of the displaced water, as stated by Archimedes' principle. Thus, we have

$$F_B = W_{H_2O} = W_P + W_J \tag{2}$$

SOLUTION Solving Equation 4.2b for the acceleration a_y gives

$$a_y = \frac{F_B - W}{m} \qquad (1)$$

The weight W of an object is equal to its mass m times the magnitude g of the acceleration due to gravity, or $W = mg$ (Equation 4.5). The mass, in turn, is equal to the product of an object's density ρ and its volume V, so $m = \rho V$ (Equation 11.1). Combining these two relations, the weight can be expressed as $W = \rho V g$.

According to Archimedes' principle, the magnitude F_B of the buoyant force is equal to the weight of the cool air that the balloon displaces, so $F_B = m_{\text{cool air}} g = (\rho_{\text{cool air}} V)g$. Since we are neglecting the weight of the balloon fabric and the basket, the weight of the balloon is just that of the hot air inside the balloon. Thus, $m = m_{\text{hot air}} = \rho_{\text{hot air}} V$ and $W = m_{\text{hot air}} g = (\rho_{\text{hot air}} V)g$.

Substituting the expressions $F_B = (\rho_{\text{cool air}} V)g$, $m = \rho_{\text{hot air}} V$, and $W = (\rho_{\text{hot air}} V)g$ into Equation (1) gives

$$a_y = \frac{F_B - W}{m} = \frac{\rho_{\text{cool air}} Vg - \rho_{\text{hot air}} Vg}{\rho_{\text{hot air}} V} = \frac{\left(\rho_{\text{cool air}} - \rho_{\text{hot air}}\right) g}{\rho_{\text{hot air}}}$$

$$= \frac{\left(1.29 \text{ kg/m}^3 - 0.93 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)}{0.93 \text{ kg/m}^3} = \boxed{3.8 \text{ m/s}^2}$$

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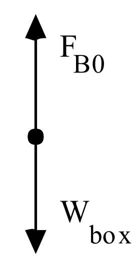
49. ***REASONING AND SOLUTION*** The figure at the right shows the two forces that initially act on the box. Since the box is not accelerated, the two forces must have zero resultant: $F_{B0} - W_{\text{box}} = 0$. Therefore,



$$F_{B0} = W_{\text{box}} \qquad (1)$$

From Archimedes' principle, the buoyant force on the box is equal to the weight of the water that is displaced by the box:

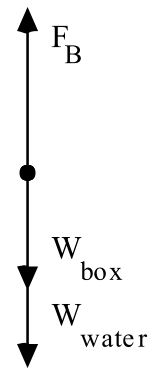
$$F_{B0} = W_{\text{disp}} \qquad (2)$$

Combining (1) and (2) we have $W_{\text{box}} = W_{\text{disp}}$, or $m_{\text{box}}\, g = \rho_{\text{water}} g V_{\text{disp}}$. Therefore,

$$m_{\text{box}} = \rho_{\text{water}} V_{\text{disp}}$$

Since the box floats with one-third of its height beneath the water, $V_{\text{disp}} = (1/3)V_{\text{box}}$, or $V_{\text{disp}} = (1/3)L^3$. Therefore,

$$m_{\text{box}} = \frac{\rho_{\text{water}} L^3}{3} \qquad (3)$$



The figure at the right shows the three forces that act on the box after water is poured into the box. The box begins to sink when

$$W_{\text{box}} + W_{\text{water}} \geq F_{\text{B}} \qquad (4)$$

The box just begins to sink when the equality is satisfied. From Archimedes' principle, the buoyant force on the system is equal to the weight of the water that is displaced by the system: $F_{\text{B}} = W_{\text{displaced}}$.

The equality in Equation (4) can be written as

$$m_{\text{box}}g + m_{\text{water}}g = m_{\text{displaced}}g \qquad (5)$$

When the box begins to sink, the volume of the water displaced is equal to the volume of the box; Equation (5) then becomes

$$m_{\text{box}} + \rho_{\text{water}}V_{\text{water}} = \rho_{\text{water}}V_{\text{box}}.$$

The volume of water in the box at this instant is $V_{\text{water}} = L^2h$, where h is the depth of the water in the box. Thus, the equation above becomes

$$m_{\text{box}} + \rho_{\text{water}}L^2h = \rho_{\text{water}}L^3$$

Using Equation (3) for the mass of the box, we obtain

$$\frac{\rho_{\text{water}}L^3}{3} + \rho_{\text{water}}L^2h = \rho_{\text{water}}L^3$$

Solving for h gives

$$h = \tfrac{2}{3}L = \tfrac{2}{3}(0.30\text{ m}) = \boxed{0.20\text{ m}}$$

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50. ***REASONING*** The fraction of the specimen's apparent volume that is solid will be given by

$$\frac{V_{\text{solid}}}{V_{\text{total}}} = \frac{V_{\text{solid}}}{V_{\text{solid}} + V_{\text{hollow}}} \qquad (1)$$

where V_{solid} is the volume of the solid part of the specimen and V_{hollow} is the volume of the hollow part. Since the specimen weighs twice as much in air as it does in water, $W_{\text{air}} = 2W_{\text{water}}$. Furthermore, we are told that the density of the solid part of the specimen is $5.0 \times 10^3\text{ kg/m}^3$; this is five times greater than the density of water ρ_{fluid}, so that $\rho_{\text{solid}} = 5\rho_{\text{fluid}}$. Using this information and Archimedes' principle, we will obtain a value for the fraction expressed in Equation (1).

SOLUTION Let M represent the mass of the rock. Then, according to Equation 11.1, the volume of the solid part of the rock is $V_{\text{solid}} = M / \rho_{\text{solid}}$.

Choosing downward as the positive direction, so $y = +0.10$ m and $a = +9.80$ m/s^2, the cross-sectional area of the stream at a distance of 0.10 m below the faucet is

$$A_1 = \frac{A_2 v_2}{\sqrt{v_2^2 + 2ay}} = \frac{\left(1.8\times10^{-4}\text{ m}^2\right)(0.85\text{ m/s})}{\sqrt{(0.85\text{ m/s})^2 + 2\left(9.80\text{ m/s}^2\right)(0.10\text{ m})}} = \boxed{9.3\times10^{-5}\text{ m}^2}$$

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59. ***REASONING*** The number N of capillaries can be obtained by dividing the total cross-sectional area A_{cap} of all the capillaries by the cross-sectional area a_{cap} of a single capillary. We know the radius r_{cap} of a single capillary, so a_{cap} can be calculated as $a_{cap} = \pi r_{cap}^2$. To find A_{cap}, we will use the equation of continuity.

SOLUTION The number N of capillaries is

$$N = \frac{A_{cap}}{a_{cap}} = \frac{A_{cap}}{\pi r_{cap}^2} \qquad (1)$$

where the cross-sectional area a_{cap} of a single capillary has been replaced by $a_{cap} = \pi r_{cap}^2$. To obtain the total cross-sectional area A_{cap} of all the capillaries, we use the equation of continuity for an incompressible fluid (see Equation 11.9). For present purposes, this equation is

$$A_{aorta} v_{aorta} = A_{cap} v_{cap} \quad \text{or} \quad A_{cap} = \frac{A_{aorta} v_{aorta}}{v_{cap}} \qquad (2)$$

The cross-sectional area of the aorta is $A_{aorta} = \pi r_{aorta}^2$, and with this substitution, Equation (2) becomes

$$A_{cap} = \frac{A_{aorta} v_{aorta}}{v_{cap}} = \frac{\pi r_{aorta}^2 v_{aorta}}{v_{cap}}$$

Substituting this result into Equation (1), we find that

$$N = \frac{A_{cap}}{\pi r_{cap}^2} = \frac{\left(\dfrac{\pi r_{aorta}^2}{v_{cap}}\right) v_{aorta}}{\pi r_{cap}^2} = \frac{r_{aorta}^2 v_{aorta}}{r_{cap}^2 v_{cap}}$$

$$= \frac{(1.1\text{ cm})^2 (40\text{ cm/s})}{\left(6\times10^{-4}\text{ cm}\right)^2 (0.07\text{ cm/s})} = \boxed{2\times10^9}$$

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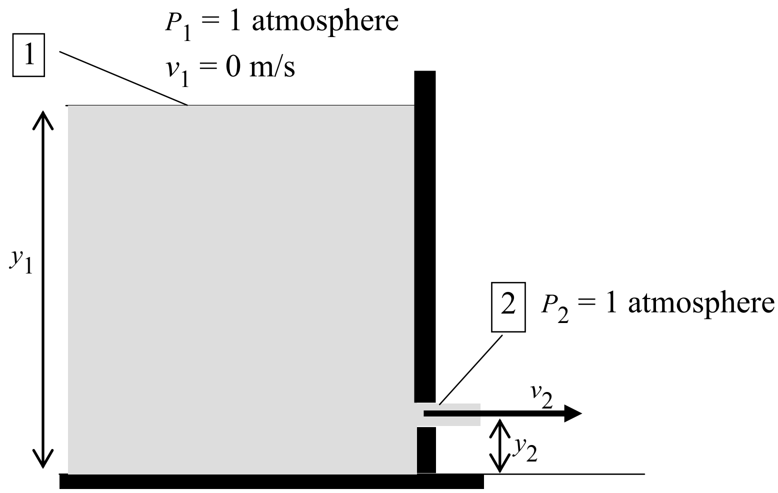
66. ***REASONING***

a. The drawing shows two points, labeled 1 and 2, in the fluid. Point 1 is at the top of the water, and point 2 is where it flows out of the dam at the bottom. Bernoulli's equation, Equation 11.11, can be used to determine the speed v_2 of the water exiting the dam.



b. The number of cubic meters per second of water that leaves the dam is the volume flow rate Q. According to Equation 11.10, the volume flow rate is the product of the cross-sectional area A_2 of the crack and the speed v_2 of the water; $Q = A_2v_2$.

SOLUTION

a. According to Bernoulli's equation, as given in Equation 11.11, we have

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Setting $P_1 = P_2$, v_1 = 0 m/s, and solving for v_2, we obtain

$$v_2 = \sqrt{2g\left(y_1 - y_2\right)} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(15.0\text{ m}\right)} = \boxed{17.1\text{ m/s}}$$

b. The volume flow rate of the water leaving the dam is

$$Q = A_2v_2 = \left(1.30\times10^{-3}\text{m}^2\right)\left(17.1\text{ m/s}\right) = \boxed{2.22\times10^{-2}\text{ m}^3\text{/s}} \qquad (11.10)$$

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67. ***REASONING*** The pressure P, the fluid speed v, and the elevation y at any two points in an ideal fluid of density ρ are related by Bernoulli's equation: $P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$ (Equation 11.11), where 1 and 2 denote, respectively, the first and second floors. With the given data and a density of $\rho = 1.00 \times 10^3$ kg/m^3 for water (see Table 11.1), we can solve Bernoulli's equation for the desired pressure P_2.

SOLUTION

Solving Bernoulli's equation for P_2 and taking the elevation at the first floor to be $y_1 = 0$ m, we have

$$P_2 = P_1 + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

$$= 3.4\times10^5\ \text{Pa} + \tfrac{1}{2}\left(1.00\times10^3\ \text{kg/m}^3\right)\left[(2.1\ \text{m/s})^2 - (3.7\ \text{m/s})^2\right]$$

$$+\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(0\ \text{m} - 4.0\ \text{m}\right) = \boxed{3.0\times10^5\ \text{Pa}}$$

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68. ***REASONING*** The volume of water per second leaking into the hold is the volume flow rate Q. The volume flow rate is the product of the effective area A of the hole and the speed v_1 of the water entering the hold, $Q = Av_1$ (Equation 11.10). We can find the speed v_1 with the aid of Bernoulli's equation.

SOLUTION According to Bernoulli's equation, which relates the pressure P, water speed v, and elevation y of two points in the water:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad (11.11)$$

In this equation, the subscript "1" refers to the point below the surface where the water enters the hold, and the subscript "2" refers to a point on the surface of the lake. Since the amount of water in the lake is large, the water level at the surface drops very, very slowly as water enters the hold of the ship. Thus, to a very good approximation, the speed of the water at the surface is zero, so $v_2 = 0$ m/s. Setting $P_1 = P_2$ (since the empty hold is open to the atmosphere) and $v_2 = 0$ m/s, and then solving for v_1, we obtain $v_1 = \sqrt{2g\left(y_2 - y_1\right)}$. Substituting $v_1 = \sqrt{2g\left(y_2 - y_1\right)}$ into Equation 11.10, we find that the volume flow rate Q of the water entering the hold is

$$Q = Av_1 = A\sqrt{2g\left(y_2 - y_1\right)} = \left(8.0\times10^{-3}\text{m}^2\right)\sqrt{2\left(9.80\ \text{m/s}^2\right)(2.0\ \text{m})} = \boxed{5.0\times10^{-2}\ \text{m}^3/\text{s}}$$

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69. SSM ***REASONING*** Since the pressure difference is known, Bernoulli's equation can be used to find the speed v_2 of the gas in the pipe. Bernoulli's equation also contains the unknown speed v_1 of the gas in the Venturi meter; therefore, we must first express v_1 in terms of v_2. This can be done by using Equation 11.9, the equation of continuity.

SOLUTION

a. From the equation of continuity (Equation 11.9) it follows that $v_1 = \left(A_2 / A_1\right)v_2$. Therefore,

$$v_1 = \frac{0.0700\ \text{m}^2}{0.0500\ \text{m}^2}v_2 = (1.40)v_2$$

Substituting this expression into Bernoulli's equation (Equation 11.12), we have

$$P_1 + \tfrac{1}{2}\rho(1.40\ v_2)^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Solving for v_2, we obtain

$$v_2 = \sqrt{\frac{2(P_2 - P_1)}{\rho\ \left[(1.40)^2 - 1\right]}} = \sqrt{\frac{2(120\ \text{Pa})}{(1.30\ \text{kg/m}^3)\ \left[(1.40)^2 - 1\right]}} = \boxed{14\ \text{m/s}}$$

b. According to Equation 11.10, the volume flow rate is

$$Q = A_2 v_2 = (0.0700\ \text{m}^2)(14\ \text{m/s}) = \boxed{0.98\ \text{m}^3/\text{s}}$$

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70. ***REASONING*** The gauge pressure in the reservoir is the pressure difference $P_2 - P_1$ between the reservoir (P_2) and the atmosphere (P_1). The muzzle is open to the atmosphere, so the pressure there is atmospheric pressure. Because we are ignoring the height difference between the reservoir and the muzzle, this is an example of fluid flow in a horizontal pipe, for which Bernoulli's equation is

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \qquad (11.12)$$

Solving Equation 11.12 for the gauge pressure $P_2 - P_1$ yields

$$P_2 - P_1 = \tfrac{1}{2}\rho v_1^2 - \tfrac{1}{2}\rho v_2^2 = \tfrac{1}{2}\rho v_1^2 \qquad (1)$$

where we have used the fact that the speed v_2 of the water in the reservoir is zero. We will determine the speed v_1 of the water stream at the muzzle by considering its subsequent projectile motion. The horizontal displacement of the water stream after leaving the muzzle is given by $x = v_{0x}t + \tfrac{1}{2}a_x t^2$ (Equation 3.5a). With air resistance neglected, the water stream undergoes no horizontal acceleration (a_x = 0 m/s^2). The horizontal component v_{0x} of the initial velocity of the water stream is identical with the velocity v_1 in Equation (1), so Equation 3.5a becomes

$$x = v_1 t + \tfrac{1}{2}\left(0\ \text{m/s}^2\right)t^2 \qquad \text{or} \qquad v_1 = \frac{x}{t} \qquad (2)$$

The vertical displacement y of the water stream is given by $y = v_{0y}t + \tfrac{1}{2}a_y t^2$ (Equation 3.5b), where a_y = −9.8 m/s^2 is the acceleration due to gravity, with upward taken as the positive direction. The velocity of the water at the instant it leaves the muzzle is horizontal, so the vertical component of its velocity is zero. This means that we have v_{0y} = 0 m/s in Equation 3.5b. Solving Equation 3.5b for the elapsed time t, we obtain

$$y = \left(0\ \text{m/s}\right)t + \tfrac{1}{2}a_y t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2y}{a_y}} \qquad (3)$$

Solutions for Home work Problems

Temperature and Heat: Chapter 12

CHAPTER 12 | *TEMPERATURE AND HEAT*

PROBLEMS

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1. SSM ***REASONING***

a. According to the discussion in Section 12.1, the size of a Fahrenheit degree is smaller than that of a Celsius degree by a factor of $\frac{5}{9}$; thus, $1\ \text{F}° = \frac{5}{9}\ \text{C}°$. This factor will be used to find the temperature difference in Fahrenheit degrees.

b. The size of one kelvin is identical to that of one Celsius degree (see Section 12.2), 1 K = 1 C°. Thus, the temperature difference, expressed in kelvins, is the same at that expressed in Celsius degrees.

SOLUTION

a. The difference in the two temperatures is $34\ °\text{C} - 3\ °\text{C} = 31\ \text{C}°$. This difference, expressed in Fahrenheit degrees, is

$$\text{Temperature difference} = 31\ \text{C}° = \left(31\ \cancel{\text{C}°}\right)\left(\frac{1\ \text{F}°}{\frac{5}{9}\ \cancel{\text{C}°}}\right) = \boxed{56\ \text{F}°}$$

b. Since 1 K = 1 C°, the temperature difference, expressed in kelvins, is

$$\text{Temperature difference} = 31\ \text{C}° = \left(31\ \cancel{\text{C}°}\right)\left(\frac{1\ \text{K}}{1\ \cancel{\text{C}°}}\right) = \boxed{31\ \text{K}}$$

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2. ***REASONING*** We will first convert the temperature from the Kelvin scale to the Celsius scale by using Equation 12.1, $T_c = T - 273.15$, where T_c is the Celsius temperature and T is the Kelvin temperature. Then, following the approach discussed in Section 12.1, we will convert from the Celsius scale to the Fahrenheit scale by multiplying the Celsius temperature by a factor of 9/5 and adding 32 to the result.

SOLUTION The temperature on the Celsius scale is

$$T_c = T - 273.15 = (312.0 - 273.15)\ °\text{C} = 38.9\ °\text{C} \qquad (12.1)$$

The temperature of 38.9 °C is equivalent to a Fahrenheit temperature of

$$\text{Temperature} = (38.9)\left(\frac{9}{5}\right) + 32.0 = \boxed{102\ °\text{F}}$$

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10. ***REASONING*** The change in length ΔL of the pipe is proportional to the coefficient of linear expansion α for steel, the original length L_0 of the pipe, and the change in temperature ΔT. The coefficient of linear expansion for steel can be found in Table 12.1.

SOLUTION The change in length of the pipe is

$$\Delta L = \alpha L_0 \Delta T = \left[1.2\times10^{-5}\,(\text{C}°)^{-1}\right](65\text{ m})\left[18\,°\text{C} - (-45\,°\text{C})\right] = \boxed{4.9\times10^{-2}\text{ m}} \qquad (12.2)$$

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11. SSM ***REASONING AND SOLUTION*** Using Equation 12.2 and the value for the coefficient of thermal expansion of steel given in Table 12.1, we find that the linear expansion of the aircraft carrier is

$$\Delta L = \alpha L_0\,\Delta T = (12\times10^{-6}\text{ C}°^{-1})(370\text{ m})(21\,°\text{C} - 2.0\,°\text{C}) = \boxed{0.084\text{ m}} \qquad (12.2)$$

---

12. ***REASONING*** The height L_0 of the Eiffel Tower at the lower temperature can be determined from $L_0 = \Delta L/(\alpha\Delta T)$ (Equation 12.2), where ΔL is the increase in the height, α is the coefficient of linear expansion for steel, and ΔT is the change in temperature. The coefficient of linear expansion for steel can be found in Table 12.1.

SOLUTION The height of the Eiffel Tower at the lower temperature is

$$L_0 = \frac{\Delta L}{\alpha\Delta T} = \frac{19.4\times10^{-2}\text{ m}}{\left[1.2\times10^{-5}\,(\text{C}°)^{-1}\right]\left[41\,°\text{C} - (-9\,°\text{C})\right]} = \boxed{3.2\times10^{2}\text{ m}} \qquad (12.2)$$

---

13. ***REASONING AND SOLUTION***

a. The radius of the hole will be $\boxed{\text{larger}}$ when the plate is heated, because the hole expands as if it were made of copper.

b. According to Equation 12.2, the expansion of the radius is $\Delta r = \alpha r_0 \Delta T$. Using the value for the coefficient of thermal expansion of copper given in Table 12.1, we find that the fractional change in the radius is

$$\Delta r/r_0 = \alpha\,\Delta T = [17\times10^{-6}\,(\text{C}°)^{-1}](110\,°\text{C} - 11\,°\text{C}) = \boxed{0.0017}$$

---

14. ***REASONING AND SOLUTION*** The value for the coefficient of thermal expansion of steel is given in Table 12.1. The relation, $\Delta L = \alpha L_0 \Delta T$, written in terms of the diameter d of the rod, is

$$\Delta T = \frac{\Delta d}{\alpha d_0} = \frac{0.0026\ \text{cm}}{\left[12\times10^{-6}\ (\text{C}°)^{-1}\right](2.0026\ \text{cm})} = \boxed{110\ \text{C}°} \qquad (12.2)$$

---

15. SSM ***REASONING AND SOLUTION*** The change in the coin's diameter is $\Delta d = \alpha d_0 \Delta T$, according to Equation 12.2. Solving for α gives

$$\alpha = \frac{\Delta d}{d_0\,\Delta T} = \frac{2.3\times10^{-5}\ \text{m}}{(1.8\times10^{-2}\ \text{m})(75\ \text{C}°)} = \boxed{1.7\times10^{-5}\ (\text{C}°)^{-1}} \qquad (12.2)$$

---

16. ***REASONING*** The average speed $\bar{v}$ of the flagpole's contraction is given by $\bar{v} = \frac{\Delta L}{\Delta t}$ (Equation 2.1), where ΔL is the amount by which it contracts, and Δt is the elapsed time. The amount of contraction the pole undergoes is found from $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), where α is the coefficient of thermal expansion for aluminum (see Table 12.1 in the text), L_0 is the length of the pole before it begins contracting, and ΔT is the difference between the higher and lower temperatures of the pole.

SOLUTION Substituting $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2) into $\bar{v} = \frac{\Delta L}{\Delta t}$ (Equation 2.1) yields

$$\bar{v} = \frac{\alpha L_0 \Delta T}{\Delta t} \qquad (1)$$

The elapsed time Δt is given in minutes, which must be converted to SI units (seconds) before employing Equation (1):

$$\Delta t = (27.0\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 1620\ \text{s}$$

Substituting this result and the given values into Equation (1), we obtain

$$\bar{v} = \frac{\left[23\times10^{-6}\ (\text{C}°)^{-1}\right](19\ \text{m})\left[12.0\ °\text{C} - (-20.0\ °\text{C})\right]}{1620\ \text{s}} = \boxed{8.6\times10^{-6}\ \text{m/s}}$$

---

17. ***REASONING*** According to $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), the factors that determine the amount ΔL by which the length of a rod changes are the coefficient of thermal expansion α, its initial length L_0, and the change ΔT in temperature. The materials from which the rods are made have different coefficients of thermal expansion (see Table 12.1 in the text). Also,

the change in length is the same for each rod when the change in temperature is the same. Therefore, the initial lengths must be different to compensate for the fact that the expansion coefficients are different.

SOLUTION The change in length of the lead rod is (from Equation 12.2)

$$\Delta L_{\text{L}} = \alpha_{\text{L}} L_{0,\text{ L}} \Delta T \tag{1}$$

Similarly, the change in length of the quartz rod is

$$\Delta L_{\text{Q}} = \alpha_{\text{Q}} L_{0,\text{ Q}} \Delta T \tag{2}$$

In Equations (1) and (2) the temperature change ΔT is the same for both rods. Since each changes length by the same amount, $\Delta L_{\text{L}} = \Delta L_{\text{Q}}$. Equating Equations (1) and (2) and solving for $L_{0,\text{ Q}}$ yields

$$L_{0,\text{ Q}} = \left(\frac{\alpha_{\text{L}}}{\alpha_{\text{Q}}}\right) L_{0,\text{ L}} = \left[\frac{29\times10^{-6}\,(\text{C}°)^{-1}}{0.50\times10^{-6}\,(\text{C}°)^{-1}}\right](0.10\text{ m}) = \boxed{5.8\text{ m}}$$

The values for the coefficients of thermal expansion for lead and quartz have been taken from Table 12.1.

---

18. ***REASONING*** To determine the fractional decrease in length $\dfrac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}}$, we need the decrease ΔL in the rod's length. It is the sum of the decreases in the silver part and the gold part of the rod, or $\Delta L = \Delta L_{\text{Silver}} + \Delta L_{\text{Gold}}$. Each of the decreases can be expressed in terms of the coefficient of linear expansion α, the initial length L_0, and the change in temperature ΔT, according to Equation 12.2.

SOLUTION Using Equation 12.2 to express the decrease in length of each part of the rod, we find the total decrease in the rod's length to be

$$\Delta L = \underbrace{\alpha_{\text{Silver}} L_{0,\text{ Silver}} \Delta T}_{\Delta L_{\text{Silver}}} + \underbrace{\alpha_{\text{Gold}} L_{0,\text{ Gold}} \Delta T}_{\Delta L_{\text{Gold}}}$$

The fractional decrease in the rod's length is, then,

$$\frac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}} = \frac{\alpha_{\text{Silver}} L_{0,\text{ Silver}} \Delta T + \alpha_{\text{Gold}} L_{0,\text{ Gold}} \Delta T}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}}$$

$$= \alpha_{\text{Silver}} \underbrace{\left(\frac{L_{0,\text{ Silver}}}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}}\right)}_{\text{Silver fraction} = \frac{1}{3}} \Delta T + \alpha_{\text{Gold}} \underbrace{\left(\frac{L_{0,\text{ Gold}}}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}}\right)}_{\text{Gold fraction} = \frac{2}{3}} \Delta T$$

Recognizing that one third of the rod is silver and two thirds is gold and taking values for the coefficients of linear expansion for silver and gold from Table 12.1, we have

$$\frac{\Delta L}{L_{0,\text{Silver}} + L_{0,\text{ Gold}}} = \alpha_{\text{Silver}}\left(\frac{1}{3}\right)\Delta T + \alpha_{\text{Gold}}\left(\frac{2}{3}\right)\Delta T = \left[\alpha_{\text{Silver}}\left(\frac{1}{3}\right) + \alpha_{\text{Gold}}\left(\frac{2}{3}\right)\right]\Delta T$$

$$= \left\{\left[19\times10^{-6}\ (\text{C}°)^{-1}\right]\left(\frac{1}{3}\right) + \left[14\times10^{-6}\ (\text{C}°)^{-1}\right]\left(\frac{2}{3}\right)\right\}(26\ \text{C}°) = \boxed{4.1\times10^{-4}}$$

---

19. ***REASONING AND SOLUTION*** $\Delta L = \alpha L_0 \Delta T$ gives for the expansion of the aluminum

$$\Delta L_\text{A} = \alpha_\text{A} L_\text{A} \Delta T \tag{1}$$

and for the expansion of the brass

$$\Delta L_\text{B} = \alpha_\text{B} L_\text{B} \Delta T \tag{2}$$

The air gap will be closed when $\Delta L_\text{A} + \Delta L_\text{B} = 1.3\times10^{-3}$ m. Thus, taking the coefficients of thermal expansion for aluminum and brass from Table 12.1, adding Equations (1) and (2), and solving for ΔT, we find that

$$\Delta T = \frac{\Delta L_\text{A} + \Delta L_\text{B}}{\alpha_\text{A} L_\text{A} + \alpha_\text{B} L_\text{B}} = \frac{1.3\times10^{-3}\ \text{m}}{\left[23\times10^{-6}\ (\text{C}°)^{-1}\right](1.0\ \text{m}) + \left[19\times10^{-6}\ (\text{C}°)^{-1}\right](2.0\ \text{m})} = 21\,\text{C}°$$

The desired temperature is then

$$T = 28\ °\text{C} + 21\ \text{C}° = \boxed{49\ °\text{C}}$$

---

20. ***REASONING*** Young's modulus Y can be obtained from $F = Y\left(\Delta L/L_0\right)A$ (Equation 10.17), where F is the magnitude of the stretching force applied to the ruler, ΔL and L_0 are the change in length and original length, respectively, and A is the cross-sectional area. Solving for Y gives

$$Y = \frac{F L_0}{A\Delta L}$$

The change in the length of the ruler is given by $\Delta L = \alpha L_0 \Delta T$ (Equation 12.2), where α is the coefficient of linear expansion and ΔT is the amount by which the temperature changes. Substituting this expression for ΔL into the equation above for Y gives the desired result.

SOLUTION Substituting $\Delta L = \alpha L_0 \Delta T$ into $Y = F L_0 / \left(A\Delta L\right)$ and using the fact that $\Delta T = 39$ C°, we find that

Therefore, the volume V of the carbon tetrachloride at −13.0 °C is

$$V = \left(2.54\times10^{-4}\ \text{m}^3\right)\left\{1-\left[1240\times10^{-6}\ (\text{C}^\circ)^{-1}\right]\left[75.0\ ^\circ\text{C}-\left(-13.0\ ^\circ\text{C}\right)\right]\right\} = \boxed{2.26\times10^{-4}\ \text{m}^3}$$

We have taken the value for β, the coefficient of volume expansion for carbon tetrachloride, from Table 12.1 in the text.

---

31. SSM ***REASONING AND SOLUTION*** The volume V_0 of an object changes by an amount ΔV when its temperature changes by an amount ΔT; the mathematical relationship is given by Equation 12.3: $\Delta V = \beta V_0 \Delta T$. Thus, the volume of the kettle at 24 °C can be found by solving Equation 12.3 for V_0. According to Table 12.1, the coefficient of volumetric expansion for copper is $51\times10^{-6}\ (\text{C}^\circ)^{-1}$. Solving Equation 12.3 for V_0, we have

$$V_0 = \frac{\Delta V}{\beta \Delta T} = \frac{1.2\times10^{-5}\ \text{m}^3}{[51\times10^{-6}\ (\text{C}^\circ)^{-1}](100\ ^\circ\text{C} - 24\ ^\circ\text{C})} = \boxed{3.1\times10^{-3}\ \text{m}^3}$$

---

32. ***REASONING*** Consider one gallon of cider on a day when the temperature is 4.0 ºC. On a day when the temperature is 26.0 ºC the volume of this cider will be greater than one gallon due to volume thermal expansion. It will be greater by an amount $\Delta V = \beta V_0 \Delta T$ (Equation 12.3), where β is the coefficient of volume expansion, V_0 is the initial volume of one gallon, and ΔT is the change in temperature. You can sell this extra volume of cider for $ 2.00 per gallon and earn a bit of extra cash.

SOLUTION According to Equation 12.3, the change in volume of the cider is

$$\Delta V = \beta V_0 \Delta T = \left[280\ \times\ 10^{-6}\ (\text{C}^\circ)^{-1}\right](1.0\ \text{gal})\left[(26\ ^\circ\text{C}) - (4.0\ ^\circ\text{C})\right] = 6.2\ \times\ 10^{-3}\ \text{gal}$$

At a cost of two dollars per gallon, this amounts to

$$\left(6.2\ \times\ 10^{-3}\ \text{gal}\right)\left(\frac{\$\ 2.00}{1\ \text{gal}}\right) = \$\ 0.01 \qquad \text{or} \qquad \boxed{1\ \text{penny}}$$

---

33. ***REASONING AND SOLUTION*** Both the coffee and beaker expand as the temperature increases. For the expansion of the coffee (water)

$$\Delta V_\text{W} = \beta_\text{W} V_0\, \Delta T$$

For the expansion of the beaker (Pyrex glass)

$$\Delta V_\text{G} = \beta_\text{G}\, V_0\, \Delta T$$

The excess expansion of the coffee, hence the amount which spills, is

SOLUTION Applying the energy-conservation principle and using Equation 12.4 give

$$\underbrace{c_{\text{Liquid}} m \Delta T_{\text{Liquid}}}_{\text{Heat gained by liquid}} = \underbrace{c_{\text{Glass}} m \Delta T_{\text{Glass}}}_{\text{Heat lost by glass}}$$

Since it is the same for both the glass and the liquid, the mass m can be eliminated algebraically from this equation. Solving for c_{Liquid} and taking the specific heat capacity for glass from Table 12.2, we find

$$c_{\text{Liquid}} = \frac{c_{\text{Glass}} \Delta T_{\text{Glass}}}{\Delta T_{\text{Liquid}}} = \frac{\left[840\ \text{J/}\left(\text{kg}\cdot\text{C}^\circ\right)\right]\left(83.0\ \text{C}^\circ - 53.0\ \text{C}^\circ\right)}{\left(53.0\ \text{C}^\circ - 43.0\ \text{C}^\circ\right)} = \boxed{2500\ \text{J/}\left(\text{kg}\cdot\text{C}^\circ\right)}$$

---

47. ***REASONING*** The metabolic processes occurring in the person's body produce the heat that is added to the water. As a result, the temperature of the water increases. The heat Q that must be supplied to increase the temperature of a substance of mass m by an amount ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity. The increase ΔT in temperature is the final higher temperature T_f minus the initial lower temperature T_0. Hence, we will solve Equation 12.4 for the desired final temperature.

SOLUTION From Equation 12.4, we have

$$Q = cm\Delta T = cm\left(T_f - T_0\right)$$

Solving for the final temperature, noting that the heat is $Q = (3.0 \times 10^5\ \text{J/h})(0.50\ \text{h})$ and taking the specific heat capacity of water from Table 12.2, we obtain

$$T_f = T_0 + \frac{Q}{cm} = 21.00\ {}^\circ\text{C} + \frac{\left(3.0\times10^5\ \text{J/h}\right)\left(0.50\ \text{h}\right)}{\left[4186\ \text{J/}\left(\text{kg}\cdot\text{C}^\circ\right)\right]\left(1.2\times10^3\ \text{kg}\right)} = \boxed{21.03\ {}^\circ\text{C}}$$

---

48. ***REASONING*** The change ΔT in temperature is determined by the amount Q of heat added, the specific heat capacity c and mass m of the material, according to $Q = cm\Delta T$ (Equation 12.4). The heat supplied to each bar and the mass of each bar are the same, but the changes in temperature are different. The only factor that can account for the different temperature changes is the specific heat capacities, which must be different. We will apply Equation 12.4 to each bar and thereby determine the unknown specific heat capacity.

SOLUTION The heat supplied to each bar is given by $Q = cm\Delta T$ (Equation 12.4). The amount of heat Q_G supplied to the glass is equal to the heat Q_S supplied to the other substance. Thus,

$$Q_G = Q_S \qquad \text{or} \qquad c_G\, m\Delta T_G = c_S\, m\Delta T_S \tag{1}$$

$$Q_{\text{total}} = (0.45\text{ kg})\left\{[9.00\times10^{2}\text{ J/(kg}\cdot\text{C°)}](660\text{ °C}-130\text{ °C})+4.0\times10^{5}\text{ J/kg}\right\} = \boxed{3.9\times10^{5}\text{ J}}$$

---

58. ***REASONING*** The amount of heat removed when a liquid freezes into a solid is determined by its latent heat of fusion L_f. The amount of heat removed is $Q = mL_f$ (Equation 12.5), where m is the mass of the material that freezes. The latent heat of fusion for water is 33.5×10^4 J/kg, whereas the value for ethyl alcohol is 10.8×10^4 J/kg, as given in Table 12.3.

SOLUTION The same amount of heat is removed from the water as from the ethyl alcohol. Therefore, applying Equation 12.5 to each material, we have

$$Q_{\text{water}} = Q_{\text{alcohol}} \qquad \text{or} \qquad m_{\text{water}}L_{\text{f, water}} = m_{\text{alcohol}}L_{\text{f, alcohol}}$$

Solving this expression for m_{alcohol} gives

$$m_{\text{alcohol}} = m_{\text{water}}\frac{L_{\text{f, water}}}{L_{\text{f, alcohol}}} = (3.0\text{ kg})\left(\frac{33.5\times10^{4}\text{ J/kg}}{10.8\times10^{4}\text{ J/kg}}\right) = \boxed{9.3\text{ kg}}$$

---

59. ***REASONING***

a. When water changes from the liquid to the ice phase at 0 °C, the amount of heat released is given by $Q = mL_f$ (Equation 12.5), where m is the mass of the water and L_f is its latent heat of fusion.

b. When heat Q is supplied to the tree, its temperature changes by an amount ΔT. The relation between Q and ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity and m is the mass. This equation can be used to find the change in the tree's temperature.

SOLUTION

a. Taking the latent heat of fusion for water as $L_f = 33.5 \times 10^4$ J/kg (see Table 12.3), we find that the heat released by the water when it freezes is

$$Q = mL_f = (7.2\text{ kg})(33.5\times10^{4}\text{ J/kg}) = \boxed{2.4\times10^{6}\text{ J}}$$

b. Solving Equation 12.4 for the change in temperature, we have

$$\Delta T = \frac{Q}{cm} = \frac{2.4\times10^{6}\text{ J}}{\left[2.5\times10^{3}\text{ J/(kg}\cdot\text{C°)}\right](180\text{ kg})} = \boxed{5.3\text{ C°}}$$

---

60. ***REASONING***

Solutions for Home work Problems

The Ideal Gas Law and Kinetic Theory: Chapter 14

5. SSM ***REASONING*** The mass (in grams) of the active ingredient in the standard dosage is the number of molecules in the dosage times the mass per molecule (in grams per molecule). The mass per molecule can be obtained by dividing the molecular mass (in grams per mole) by Avogadro's number. The molecular mass is the sum of the atomic masses of the molecule's atomic constituents.

SOLUTION Using N to denote the number of molecules in the standard dosage and m_{molecule} to denote the mass of one molecule, the mass (in grams) of the active ingredient in the standard dosage can be written as follows:

$$m = Nm_{\text{molecule}}$$

Using M to denote the molecular mass (in grams per mole) and recognizing that $m_{\text{molecule}} = \frac{M}{N_A}$, where N_A is Avogadro's number and is the number of molecules per mole, we have

$$m = Nm_{\text{molecule}} = N\left(\frac{M}{N_A}\right)$$

M (in grams per mole) is equal to the molecular mass in atomic mass units. We can obtain this quantity by referring to the periodic table on the inside of the back cover of the text to find the molecular masses of the constituent atoms in the active ingredient. Thus, we have

$$\text{Molecular mass} = \underbrace{22(12.011\text{ u})}_{\text{Carbon}} + \underbrace{23(1.00794\text{ u})}_{\text{Hydrogen}} + \underbrace{1(35.453\text{ u})}_{\text{Chlorine}} + \underbrace{2(14.0067\text{ u})}_{\text{Nitrogen}} + \underbrace{2(15.9994\text{ u})}_{\text{Oxygen}}$$

$$= 382.89\text{ u}$$

The mass of the active ingredient in the standard dosage is

$$m = N\left(\frac{M}{N_A}\right) = \left(1.572\times10^{19}\text{ molecules}\right)\left(\frac{382.89\text{ g/mol}}{6.022\times10^{23}\text{ molecules/mol}}\right) = \boxed{1.00\times10^{-2}\text{ g}}$$

---

6. ***REASONING*** To find the molecular mass of chlorophyll-*a* ($C_{55}H_{72}MgN_4O_5$) in atomic mass units, we use the number and atomic mass of each constituent atom in the molecule (see the periodic table of the elements on the inside of the back cover of the text). The molecular mass obtained in this fashion is the mass per mole in g/mol, which we can then use to obtain the mass (in grams) of 3.00 moles of chlorophyll-*a*.

SOLUTION

a. The molecular mass of $C_{55}H_{72}MgN_4O_5$ is

11. SSM ***REASONING*** Both gases fill the balloon to the same pressure P, volume V, and temperature T. Assuming that both gases are ideal, we can apply the ideal gas law $PV = nRT$ to each and conclude that the same number of moles n of each gas is needed to fill the balloon. Furthermore, the number of moles can be calculated from the mass m (in grams) and the mass per mole M (in grams per mole), according to $n = \frac{m}{M}$. Using this expression in the equation $n_{\text{Helium}} = n_{\text{Nitrogen}}$ will allow us to obtain the desired mass of nitrogen.

SOLUTION Since the number of moles of helium equals the number of moles of nitrogen, we have

$$\underbrace{\frac{m_{\text{Helium}}}{M_{\text{Helium}}}}_{\substack{\text{Number of moles}\\ \text{of helium}}} = \underbrace{\frac{m_{\text{Nitrogen}}}{M_{\text{Nitrogen}}}}_{\substack{\text{Number of moles}\\ \text{of nitrogen}}}$$

Solving for m_{Nitrogen} and taking the values of mass per mole for helium (He) and nitrogen (N_2) from the periodic table on the inside of the back cover of the text, we find

$$m_{\text{Nitrogen}} = \frac{M_{\text{Nitrogen}} m_{\text{Helium}}}{M_{\text{Helium}}} = \frac{(28.0 \text{ g/mol})(0.16 \text{ g})}{4.00 \text{ g/mol}} = \boxed{1.1 \text{ g}}$$

---

12. ***REASONING*** The pressure P of the water vapor in the container can be found from the ideal gas law, Equation 14.1, as $P = nRT/V$, where n is the number of moles of water, R is the universal gas constant, T is the Kelvin temperature, and V is the volume. The variables R, T, and V are known, and the number of moles can be obtained by noting that it is equal to the mass m of the water divided by its mass per mole.

SOLUTION Substituting $n = m$/(Mass per mole) into the ideal gas law, we have

$$P = \frac{nRT}{V} = \frac{\left(\dfrac{m}{\text{Mass per mole}}\right)RT}{V}$$

The mass per mole (in g/mol) of water (H_2O) has the same numerical value as its molecular mass. The molecular mass of water is 2 (1.00794 u) + 15.9994 u = 18.0153 u. The mass per mole of water is 18.0153 g/mol. Thus, the pressure of the water vapor is

$$P = \frac{\left(\dfrac{m}{\text{Mass per mole}}\right)RT}{V} = \frac{\left(\dfrac{4.0 \text{ g}}{18.0153 \text{ g/mol}}\right)[8.31 \text{ J/(mol}\cdot\text{K)}](388 \text{ K})}{0.030 \text{ m}^3} = \boxed{2.4\times10^4 \text{ Pa}}$$

---

15. SSM ***REASONING AND SOLUTION*** According to the ideal gas law (Equation 14.1), the total number of moles n of fresh air in a normal breath is

$$n = \frac{PV}{RT} = \frac{(1.0\times10^{5}\ \text{Pa})(5.0\times10^{-4}\ \text{m}^3)}{[8.31\ \text{J/(mole}\cdot\text{K)}]\ (310\ \text{K})} = 1.94\times10^{-2}\ \text{mol}$$

The total number of molecules in a normal breath is nN_A, where N_A is Avogadro's number. Since fresh air contains approximately 21% oxygen, the total number of oxygen molecules in a normal breath is $(0.21)nN_A$ or

$$(0.21)(1.94\times10^{-2}\ \text{mol})(6.022\times10^{23}\ \text{mol}^{-1}) = \boxed{2.5\times10^{21}}$$

---

16. ***REASONING*** We can use the ideal gas law, Equation 14.1 ($PV = nRT$) to find the number of moles of helium in the Goodyear blimp, since the pressure, volume, and temperature are known. Once the number of moles is known, we can find the mass of helium in the blimp.

SOLUTION The number n of moles of helium in the blimp is, according to Equation 14.1,

$$n = \frac{PV}{RT} = \frac{(1.1\times10^{5}\ \text{Pa})(5400\ \text{m}^3)}{[8.31\ \text{J/(mol}\cdot\text{K)}](280\ \text{K})} = 2.55\times10^{5}\ \text{mol}$$

According to the periodic table on the inside of the text's back cover, the atomic mass of helium is 4.002 60 u. Therefore, the mass per mole is 4.002 60 g/mol. The mass m of helium in the blimp is, then,

$$m = \left(2.55\times10^{5}\ \text{mol}\right)\left(4.002\,60\ \text{g/mol}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) = \boxed{1.0\times10^{3}\ \text{kg}}$$

---

17. ***REASONING*** The maximum number of balloons that can be filled is the volume of helium available at the pressure in the balloons divided by the volume per balloon. The volume of helium available at the pressure in the balloons can be determined using the ideal gas law. Since the temperature remains constant, the ideal gas law indicates that $PV = nRT$ = constant, and we can apply it in the form of Boyle's law, $P_iV_i = P_fV_f$. In this expression V_f is the final volume at the pressure in the balloons, V_i is the volume of the cylinder, P_i is the initial pressure in the cylinder, and P_f is the pressure in the balloons. However, we need to remember that a volume of helium equal to the volume of the cylinder will remain in the cylinder when its pressure is reduced to atmospheric pressure at the point when balloons can no longer be filled.

SOLUTION Using Boyle's law we find that

$$V_f = \frac{P_iV_i}{P_f}$$

$$P_2 = P_0 + \rho g h_2 = \left(1.01\times10^5 \text{ Pa}\right) + \left[\left(1.36\times10^4 \text{kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(0.380 \text{ m})\right] = 1.52\times10^5 \text{ Pa}$$

In these pressure calculations, the density of mercury is $\rho = 1.36\times10^4$ kg/m^3. In Equation (1) we note that $d_1 = 0.760$ m and $d_2 = 1.14$ m. Solving Equation (1) for T_2 and substituting values, we obtain

$$T_2 = \left(\frac{P_2 d_2}{P_1 d_1}\right) T_1 = \left[\frac{\left(1.52\times10^5 \text{ Pa}\right)(1.14 \text{ m})}{\left(2.02\times10^5 \text{ Pa}\right)(0.760 \text{ m})}\right](273 \text{ K}) = \boxed{308 \text{ K}}$$

---

32. ***REASONING AND SOLUTION*** At the instant just before the balloon lifts off, the buoyant force from the outside air has a magnitude that equals the magnitude of the total weight. According to Archimedes' principle, the buoyant force is the weight of the displaced outside air (density ρ_0 = 1.29 kg/m^3). The mass of the displaced outside air is $\rho_0 V$, where V = 650 m^3. The corresponding weight is the mass times the magnitude g of the acceleration due to gravity. Thus, we have

$$\underbrace{\left(\rho_0 V\right) g}_{\text{Buoyant force}} = \underbrace{m_{\text{total}} g}_{\text{Total weight of balloon}} \qquad (1)$$

The total mass of the balloon is $m_{\text{total}} = m_{\text{load}} + m_{\text{air}}$, where m_{load} = 320 kg and m_{air} is the mass of the hot air within the balloon. The mass of the hot air can be calculated from the ideal gas law by using it to obtain the number of moles n of air and multiplying n by the mass per mole of air, $M = 29 \times 10^{-3}$ kg/mol:

$$m_{\text{air}} = nM = \left(\frac{PV}{RT}\right)M$$

Thus, the total mass of the balloon is $m_{\text{total}} = m_{\text{load}} + PVM/(RT)$ and Equation (1) becomes

$$\rho_0 V = m_{\text{load}} + \left(\frac{PV}{RT}\right)M$$

Solving for T gives

$$T = \frac{PVM}{\left(\rho_0 V - m_{\text{load}}\right)R}$$

$$= \frac{\left(1.01\times10^5 \text{ Pa}\right)\left(650 \text{ m}^3\right)\left(29\times10^{-3} \text{ kg/mol}\right)}{\left[\left(1.29 \text{ kg/m}^3\right)\left(650 \text{ m}^3\right) - 320 \text{ kg}\right]\left[8.31 \text{ J/}\left(\text{mol}\cdot\text{K}\right)\right]} = \boxed{440 \text{ K}}$$

---

33. **SSM** ***REASONING*** The smoke particles have the same average translational kinetic energy as the air molecules, namely, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$, according to Equation 14.6. In this expression m is the mass of a smoke particle, v_{rms} is the rms speed of a particle, k is

Boltzmann's constant, and T is the Kelvin temperature. We can obtain the mass directly from this equation.

SOLUTION Solving Equation 14.6 for the mass m, we find

$$m = \frac{3kT}{v_{\text{rms}}^2} = \frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(301\ \text{K}\right)}{\left(2.8\times10^{-3}\ \text{m/s}\right)^2} = \boxed{1.6\times10^{-15}\ \text{kg}}$$

---

34. ***REASONING*** According to the kinetic theory of gases, the average kinetic energy of an atom is related to the temperature of the gas by $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$ (Equation 14.6). We see that the temperature is proportional to the product of the mass and the square of the rms-speed. Therefore, the tank with the greatest value of mv_{rms}^2 has the greatest temperature. Using the information from the table given with the problem statement, we see that the values of mv_{rms}^2 for each tank are:

Tank	Product of the mass and the square of the rms-speed
A	mv_{rms}^2
B	$m\left(2v_{\text{rms}}\right)^2 = 4mv_{\text{rms}}^2$
C	$\left(2m\right)v_{\text{rms}}^2 = 2mv_{\text{rms}}^2$
D	$2m\left(2v_{\text{rms}}\right)^2 = 8mv_{\text{rms}}^2$

Thus, tank D has the greatest temperature, followed by tanks B, C, and A.

SOLUTION The temperature of the gas in each tank can be determined from Equation 14.6:

$$T_{\text{A}} = \frac{mv_{\text{rms}}^2}{3k} = \frac{\left(3.32\times10^{-26}\ \text{kg}\right)\left(1223\ \text{m/s}\right)^2}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{1200\ \text{K}}$$

$$T_{\text{B}} = \frac{m\left(2v_{\text{rms}}\right)^2}{3k} = \frac{\left(3.32\times10^{-26}\ \text{kg}\right)\left(2\times1223\ \text{m/s}\right)^2}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{4800\ \text{K}}$$

$$T_{\text{C}} = \frac{\left(2m\right)v_{\text{rms}}^2}{3k} = \frac{2\left(3.32\times10^{-26}\ \text{kg}\right)\left(1223\ \text{m/s}\right)^2}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{2400\ \text{K}}$$

$$T_{\text{D}} = \frac{2m\left(2v_{\text{rms}}\right)^2}{3k} = \frac{2\left(3.32\times10^{-26}\ \text{kg}\right)\left(2\times1223\ \text{m/s}\right)^2}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{9600\ \text{K}}$$

These results confirm the conclusion reached in the ***REASONING***.

$$t = \frac{L^2}{2D} = \frac{(0.010\ \text{m})^2}{2\left(2.4\times10^{-5}\ \text{m}^2/\text{s}\right)} = \boxed{2.1\ \text{s}}$$

b. If a water molecule were traveling at the translational rms speed v_{rms} for water, the time t it would take to travel the distance $L = 0.010\ \text{m}$ would be given by $t = L/v_{\text{rms}}$, where, according to Equation 14.6 ($\overline{\text{KE}} = \frac{1}{2}mv_{\text{rms}}^2$), $v_{\text{rms}} = \sqrt{2\left(\overline{\text{KE}}\right)/m}$. Before we can use the last expression for the translation rms speed v_{rms}, we must determine the mass m of a water molecule and the average translational kinetic energy $\overline{\text{KE}}$. Using the periodic table on the inside of the text's back cover, we find that the molecular mass of a water molecule is

$$\underbrace{2(1.00794\ \text{u})}_{\substack{\text{Mass of two} \\ \text{hydrogen atoms}}} + \underbrace{15.9994\ \text{u}}_{\substack{\text{Mass of one} \\ \text{oxygen atom}}} = 18.0153\ \text{u}$$

The mass of a single molecule is

$$m = \frac{18.0153\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{mol}^{-1}} = 2.99\times10^{-26}\ \text{kg}$$

The average translational kinetic energy of water molecules at 293 K is, according to Equation 14.6,

$$\overline{\text{KE}} = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/K})(293\ \text{K}) = 6.07\times10^{-21}\ \text{J}$$

Therefore, the translational rms speed of water molecules is

$$v_{\text{rms}} = \sqrt{\frac{2\left(\overline{\text{KE}}\right)}{m}} = \sqrt{\frac{2\left(6.07\times10^{-21}\ \text{J}\right)}{2.99\times10^{-26}\ \text{kg}}} = 637\ \text{m/s}$$

Thus, the time t required for a water molecule to travel the distance $L = 0.010\ \text{m}$ at this speed is

$$t = \frac{L}{v_{\text{rms}}} = \frac{0.010\ \text{m}}{637\ \text{m/s}} = \boxed{1.6\times10^{-5}\ \text{s}}$$

c. In part (a), when a water molecule diffuses through air, it makes millions of collisions each second with air molecules. The speed and direction change abruptly as a result of each collision. Between collisions, the water molecules move in a straight line at constant speed. Although a water molecule does move very quickly between collisions, it wanders only very slowly in a zigzag path from one end of the channel to the other. In contrast, a water molecule traveling unobstructed at its translational rms speed [as in part (b)], will have a larger displacement over a much shorter time. Therefore, the answer to part (a) is much longer than the answer to part (b).

---

48. ***REASONING*** The diffusion of the glycine is driven by the concentration difference $\Delta C = C_2 - C_1$ between the ends of the tube, where C_2 is the higher concentration and

$C_1 = C_2 - \Delta C$ is the lower concentration. The concentration difference is related to the mass rate of diffusion by Fick's law: $\dfrac{m}{t} = \dfrac{DA\Delta C}{L}$ (Equation 14.8), where D is the diffusion constant of glycine in water, and A and L are, respectively, the cross-sectional area and length of the tube.

SOLUTION Solving $\dfrac{m}{t} = \dfrac{DA\Delta C}{L}$ (Equation 14.8) for ΔC, we obtain

$$\Delta C = \frac{L\left(\dfrac{m}{t}\right)}{DA} \tag{1}$$

Substituting Equation (1) into $C_1 = C_2 - \Delta C$ yields

$$C_1 = C_2 - \Delta C = C_2 - \frac{L\left(\dfrac{m}{t}\right)}{DA} \tag{2}$$

We note that the length of the tube is L = 2.0 cm = 0.020 m. Substituting this and the other given values into Equation (2), we find that the lower concentration is

$$C_1 = 8.3\times10^{-3}\ \text{kg/m}^3 - \frac{(0.020\ \text{m})\left(4.2\times10^{-14}\ \text{kg/s}\right)}{\left(1.06\times10^{-9}\ \text{m}^2\text{/s}\right)\left(1.5\times10^{-4}\ \text{m}^2\right)} = \boxed{3.0\times10^{-3}\ \text{kg/m}^3}$$

---

49. SSM ***REASONING*** The mass m of methane that diffuses out of the tank in a time t is given by $m = \dfrac{(DA\,\Delta C)t}{L}$ (Equation 14.8), where D is the diffusion constant for methane, A is the cross-sectional area of the pipe, L is the length of the pipe, and $\Delta C = C_1 - C_2$ is the difference in the concentration of methane at the two ends of the pipe. The higher concentration C_1 at the tank-end of the pipe is given, and the concentration C_2 at the end of the pipe that is open to the atmosphere is zero.

SOLUTION Solving $m = \dfrac{(DA\,\Delta C)t}{L}$ (Equation 14.8) for the cross-sectional area A, we obtain

$$A = \frac{mL}{(D\,\Delta C)t} \tag{1}$$

The elapsed time t must be converted from hours to seconds:

$$t = \left(12\ \cancel{\text{hr}}\right)\left(\frac{3600\ \text{s}}{1\ \cancel{\text{hr}}}\right) = 4.32\times10^4\ \text{s}$$

Solutions for Home work Problems

Thermodynamics: Chapter 15

Important note: These solutions are base on the equation:

$$\Delta U = Q_{added} - W_{by}$$

$$W_{by} = -W_{on}$$

In obtaining Equation (2), we have recognized that internal energy is a function only of the state of the system, so that $(\Delta U)_{\text{overall}} = 0$, because the system begins and ends in the same state. We can now substitute Equation (2) into Equation (1) and obtain that

$$W_{\text{return}} = Q_{\text{return}} - (\Delta U)_{\text{return}} = Q_{\text{return}} + (\Delta U)_{\text{outgoing}} = (-114\text{ J}) + (-147\text{ J}) = \boxed{-261\text{ J}}$$

b. Since W_{return} is negative, $\boxed{\text{work is done on the system}}$.

---

3. SSM ***REASONING*** Energy in the form of work leaves the system, while energy in the form of heat enters. More energy leaves than enters, so we expect the internal energy of the system to decrease, that is, we expect the change ΔU in the internal energy to be negative. The first law of thermodynamics will confirm our expectation. As far as the environment is concerned, we note that when the system loses energy, the environment gains it, and when the system gains energy the environment loses it. Therefore, the change in the internal energy of the environment must be opposite to that of the system.

SOLUTION
a. The system gains heat so Q is positive, according to our convention. The system does work, so W is also positive, according to our convention. Applying the first law of thermodynamics from Equation 15.1, we find for the system that

$$\Delta U = Q - W = (77\text{ J}) - (164\text{ J}) = \boxed{-87\text{ J}}$$

As expected, this value is negative, indicating a decrease.

b. The change in the internal energy of the environment is opposite to that of the system, so that $\boxed{\Delta U_{\text{environment}} = +87\text{ J}}$.

---

4. ***REASONING*** The first law of thermodynamics, which is a statement of the conservation of energy, states that, due to heat Q and work W, the internal energy of the system changes by an amount ΔU according to $\Delta U = Q - W$ (Equation 15.1). This law can be used directly to find ΔU.

SOLUTION Q is positive ($+7.6 \times 10^4$ J) since heat flows into the system; W is also positive ($+7.6 \times 10^4$ J) since work is done by the system. Using the first law of thermodynamics from Equation 15.1, we obtain

$$\Delta U = Q - W = (+7.6\times10^4\text{ J}) - (+4.8\times10^4\text{ J}) = \boxed{+2.8\times10^4\text{ J}}$$

The plus sign indicates that the internal energy of the system increases.

---

5. ***REASONING*** In both cases, the internal energy ΔU that a player loses before becoming exhausted and leaving the game is given by the first law of thermodynamics: $\Delta U = Q - W$ (Equation 15.1), where Q is the heat lost and W is the work done while playing the game. The algebraic signs of the internal energy change ΔU and the heat loss Q are negative, while that of the work W is positive.

SOLUTION
a. From the first law of thermodynamics, the work W that the first player does before leaving the game is equal to the heat lost minus the internal energy loss:

$$W = Q - \Delta U = -6.8\times10^5\ \text{J} - \left(-8.0\times10^5\ \text{J}\right) = \boxed{+1.2\times10^5\ \text{J}}$$

b. Again employing the first law of thermodynamics, we find that the heat loss Q experienced by the more-warmly-dressed player is the sum of the work done and the internal energy change:

$$Q = W + \Delta U = 2.1\times10^5\ \text{J} + \left(-8.0\times10^5\ \text{J}\right) = -5.9\times10^5\ \text{J}$$

As expected, the algebraic sign of the heat loss is negative. The magnitude of the heat loss is, therefore, $\boxed{5.9\times10^5\ \text{J}}$.

---

6. ***REASONING*** According to the discussion in Section 14.3, the internal energy U of a monatomic ideal gas is given by $U = \frac{3}{2}nRT$ (Equation 14.7), where n is the number of moles, R is the universal gas constant, and T is the Kelvin temperature. When the temperature changes to a final value of T_f from an initial value of T_i, the internal energy changes by an amount

$$\underbrace{U_f - U_i}_{\Delta U} = \tfrac{3}{2}nR\left(T_f - T_i\right)$$

Solving this equation for the final temperature yields $T_f = \left(\dfrac{2}{3nR}\right)\Delta U + T_i$. We are given n and T_i, but must determine ΔU. The change ΔU in the internal energy of the gas is related to the heat Q and the work W by the first law of thermodynamics, $\Delta U = Q - W$ (Equation 15.1). Using these two relations will allow us to find the final temperature of the gas.

SOLUTION Substituting $\Delta U = Q - W$ into the expression for the final temperature gives

$$T_f = \left(\frac{2}{3nR}\right)(Q - W) + T_i$$

$$= \left\{\frac{2}{3(3.00\ \text{mol})\left[8.31\ \text{J}/(\text{mol}\cdot\text{K})\right]}\right\}\left[+2438\ \text{J} - (-962\ \text{J})\right] + 345\ \text{K} = \boxed{436\ \text{K}}$$

9. SSM ***REASONING*** According to Equation 15.2, $W = P\Delta V$, the average pressure $\overline{P}$ of the expanding gas is equal to $\overline{P} = W / \Delta V$, where the work W done by the gas on the bullet can be found from the work-energy theorem (Equation 6.3). Assuming that the barrel of the gun is cylindrical with radius r, the volume of the barrel is equal to its length L multiplied by the area (πr^2) of its cross section. Thus, the change in volume of the expanding gas is $\Delta V = L\pi r^2$.

SOLUTION The work done by the gas on the bullet is given by Equation 6.3 as

$$W = \tfrac{1}{2} m(v_{\text{final}}^2 - v_{\text{initial}}^2) = \tfrac{1}{2}(2.6\times10^{-3}\text{ kg})[(370\text{ m/s})^2 - 0] = 180\text{ J}$$

The average pressure of the expanding gas is, therefore,

$$\overline{P} = \frac{W}{\Delta V} = \frac{180\text{ J}}{(0.61\text{ m})\pi(2.8\times10^{-3}\text{ m})^2} = \boxed{1.2\times10^7\text{ Pa}}$$

---

10. ***REASONING*** Equation 15.2 indicates that work W done at a constant pressure P is given by $W = P\Delta V$. In this expression ΔV is the change in volume; $\Delta V = V_f - V_i$, where V_f is the final volume and V_i is the initial volume. Thus, the change in volume is

$$\Delta V = \frac{W}{P} \tag{1}$$

The pressure is known, and the work can be obtained from the first law of thermodynamics as $W = Q - \Delta U$ (see Equation 15.1).

SOLUTION Substituting $W = Q - \Delta U$ into Equation (1) gives

$$\Delta V = \frac{W}{P} = \frac{Q - \Delta U}{P} = \frac{(+2780\text{ J}) - (+3990\text{ J})}{1.26\times10^5\text{ Pa}} = \boxed{-9.60\times10^{-3}\text{ m}^3}$$

Note that Q is positive (+2780 J) since the system gains heat; ΔU is also positive (+3990 J) since the internal energy of the system increases. The change ΔV in volume is negative, reflecting the fact that the final volume is less than the initial volume.

---

11. ***REASONING*** The work done in an isobaric process is given by Equation 15.2, $W = P\Delta V$; therefore, the pressure is equal to $P = W / \Delta V$. In order to use this expression, we must first determine a numerical value for the work done; this can be calculated using the first law of thermodynamics (Equation 15.1), $\Delta U = Q - W$.

SOLUTION Solving Equation 15.1 for the work W, we find

$$W = Q - \Delta U = 1500\text{ J} - (+4500\text{ J}) = -3.0\times10^3\text{ J}$$

$$W = \tfrac{3}{2}nR\left(T_\text{i} - T_\text{f}\right) = \tfrac{3}{2}(5.0\text{ mol})\left[8.31\text{ J/}(\text{mol}\cdot\text{K})\right](370\text{ K} - 290\text{ K}) = \boxed{+5.0\times10^3\text{ J}}$$

b. Since the process is adiabatic, $Q = 0$ J, and the change in the internal energy is

$$\Delta U = Q - W = 0 - 5.0\times10^3\text{ J} = \boxed{-5.0\times10^3\text{ J}}$$

---

22. ***REASONING*** When n moles of an ideal gas change quasistatically to a final volume V_f from an initial volume V_i at a constant temperature T, the work W done is (see Equation 15.3)

$$W = nRT\ln\left(\frac{V_\text{f}}{V_\text{i}}\right) \qquad \text{or} \qquad T = \frac{W}{nR\ln\left(\frac{V_\text{f}}{V_\text{i}}\right)} \tag{1}$$

where R is the universal gas constant. To determine T from Equation (1), we need a value for the work, which we do not have. However, we do have a value for the heat Q. To take advantage of this value, we note that Section 14.3 discusses the fact that the internal energy of an ideal gas is directly proportional to its Kelvin temperature. Since the temperature is constant (the neon expands isothermally), the internal energy remains constant. According to the first law of thermodynamics (Equation 15.1), the change ΔU in the internal energy is given by $\Delta U = Q - W$. Since the internal energy U is constant, $\Delta U = 0$, so that $W = Q$.

SOLUTION Substituting $W = Q$ into the expression for T in Equation (1), we find that the temperature of the gas during the isothermal expansion is

$$T = \frac{Q}{nR\ln\left(\frac{V_\text{f}}{V_\text{i}}\right)} = \frac{4.75\times10^3\text{ J}}{(3.00\text{ mol})\left[8.31\text{ J/}(\text{mol}\cdot\text{K})\right]\ln\left(\frac{0.250\text{ m}^3}{0.100\text{ m}^3}\right)} = \boxed{208\text{ K}}$$

---

23. ***REASONING*** We can use the first law of thermodynamics, $\Delta U = Q - W$ (Equation 15.1) to find the work W. The heat is $Q = -4700$ J, where the minus sign denotes that the system (the gas) loses heat. The internal energy U of a monatomic ideal gas is given by $U = \tfrac{3}{2}nRT$ (Equation 14.7), where n is the number of moles, R is the universal gas constant, and T is the Kelvin temperature. If the temperature remains constant during the process, the internal energy does not change, so $\Delta U = 0$ J.

SOLUTION The work done during the isothermal process is

$$W = Q - \Delta U = -4700\text{ J} + 0\text{ J} = \boxed{-4700\text{ J}}$$

The negative sign indicates that work is done on the system.

Using the ideal gas law, we find that

$$\frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^{\gamma} \qquad \text{becomes} \qquad \frac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\frac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^{\gamma}$$

Since $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$, the result above reduces to

$$\frac{T_{1i}}{T_{2i}} = \left(\frac{T_{1f}}{T_{2f}}\right)^{\gamma} \qquad \text{or} \qquad \frac{T_{1f}}{T_{2f}} = \left(\frac{T_{1i}}{T_{2i}}\right)^{1/\gamma} = \left(\frac{525\ \text{K}}{275\ \text{K}}\right)^{1/\gamma} = 1.474$$

Using this expression for the ratio of the final temperatures in $T_{1f} + T_{2f} = 8.00 \times 10^{2}\ \text{K}$, we find that

a. $\boxed{T_{1f} = 477\ \text{K}}$ and b. $\boxed{T_{2f} = 323\ \text{K}}$

---

34. ***REASONING*** The heat Q that must be added to n moles of a monatomic ideal gas to raise its temperature by ΔT Kelvin degrees under conditions of constant pressure is $Q = C_P n \Delta T$ (Equation 15.6), where $C_P = \frac{5}{2}R$ (Equation 15.7) is the molar specific heat capacity of a monatomic ideal gas and $R = 8.31\ \text{J}/(\text{mol}\cdot\text{K})$ is the universal gas constant.

SOLUTION Substituting Equation 15.7 into Equation 15.6 shows that the heat is

$$Q = C_P n \Delta T = \left(\tfrac{5}{2}R\right) n \Delta T \qquad (1)$$

As Section 14.1 discusses, the number of moles n is given by the mass m divided by the mass per mole:

$$n = \frac{m}{\text{Mass per mole}} = \frac{8.0\ \text{g}}{39.9\ \text{g/mol}} = 0.20\ \text{mol}$$

With this value for n, Equation (1) gives

$$Q = C_p n \Delta T = \left(\tfrac{5}{2}R\right) n \Delta T = \tfrac{5}{2}\left[8.31\ \text{J}/(\text{mol}\cdot\text{K})\right](0.20\ \text{mol})(75\ \text{K}) = \boxed{310\ \text{J}}$$

---

35. **SSM** ***REASONING AND SOLUTION*** According to the first law of thermodynamics (Equation 15.1), $\Delta U = U_f - U_i = Q - W$. Since the internal energy of this gas is doubled by the addition of heat, the initial and final internal energies are U and $2U$, respectively. Therefore,

$$\Delta U = U_f - U_i = 2U - U = U$$

Equation 15.1 for this situation then becomes $U = Q - W$. Solving for Q gives

SOLUTION Solving $Q = C_V n\,\Delta T$ (Equation 15.6) for the number n of moles of the gas, we obtain

$$n = \frac{Q}{C_V\,\Delta T} \tag{1}$$

Substituting $C_V = \frac{3}{2}R$ (Equation 15.8) and $\Delta T = T_f - T_i$ into Equation (1) yields

$$n = \frac{Q}{C_V\,\Delta T} = \frac{Q}{\left(\frac{3}{2}R\right)\left(T_f - T_i\right)} = \frac{2Q}{3R\left(T_f - T_i\right)}$$

$$= \frac{2(8500\text{ J})}{3\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](279\text{ K} - 217\text{ K})} = \boxed{11\text{ mol}}$$

---

39. ***REASONING AND SOLUTION***

a. The amount of heat needed to raise the temperature of the gas at constant volume is given by Equations 15.6 and 15.8, $Q = n\,C_V\,\Delta T$. Solving for ΔT yields

$$\Delta T = \frac{Q}{nC_v} = \frac{5.24\times10^3\text{ J}}{(3.00\text{ mol})\left(\frac{3}{2}R\right)} = \boxed{1.40\times10^2\text{ K}}$$

b. The change in the internal energy of the gas is given by the first law of thermodynamics with $W = 0$, since the gas is heated at constant volume:

$$\Delta U = Q - W = 5.24\times10^3\text{ J} - 0 = \boxed{5.24\times10^3\text{ J}}$$

c. The change in pressure can be obtained from the ideal gas law,

$$\Delta P = \frac{nR\Delta T}{V} = \frac{(3.00\text{ mol})R\left(1.40\times10^2\text{ K}\right)}{1.50\text{ m}^3} = \boxed{2.33\times10^3\text{ Pa}}$$

---

40. ***REASONING*** According to Equations 15.6 and 15.7, the heat supplied to a monatomic ideal gas at constant pressure is $Q = C_P n\Delta T$, with $C_P = \frac{5}{2}R$. Thus, $Q = \frac{5}{2}nR\,\Delta T$. The percentage of this heat used to increase the internal energy by an amount ΔU is

$$\text{Percentage} = \left(\frac{\Delta U}{Q}\right)\times100\ \% = \left(\frac{\Delta U}{\frac{5}{2}nR\,\Delta T}\right)\times100\ \% \tag{1}$$

But according to the first law of thermodynamics, $\Delta U = Q - W$. The work W is $W = P\,\Delta V$, and for an ideal gas $P\,\Delta V = nR\,\Delta T$. Therefore, the work W becomes $W = P\,\Delta V = nR\,\Delta T$ and the change in the internal energy is $\Delta U = Q - W = \frac{5}{2}nR\,\Delta T - nR\Delta T = \frac{3}{2}nR\Delta T$.

Solutions for Home work Problems

Waves and Sound : Chapter 16

CHAPTER 16 | *WAVES AND SOUND*

PROBLEMS

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1. SSM ***REASONING*** Since light behaves as a wave, its speed v, frequency f, and wavelength λ are related to according to $v = f\lambda$ (Equation 16.1). We can solve this equation for the frequency in terms of the speed and the wavelength.

SOLUTION Solving Equation 16.1 for the frequency, we find that

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{5.45 \times 10^{-7}\ \text{m}} = \boxed{5.50 \times 10^{14}\ \text{Hz}}$$

---

2. ***REASONING AND SOLUTION***

a. Since 15 boxcars pass by in 12.0 s, the boxcars pass by with a frequency of

$$f = \frac{15}{12.0\ \text{s}} = \boxed{1.25\ \text{Hz}}$$

b. Since the length of a boxcar corresponds to the wavelength λ of a wave, we have, from Equation 16.1, that

$$v = f\lambda = (1.25\ \text{Hz})(14.0\ \text{m}) = \boxed{17.5\ \text{m/s}}$$

---

3. ***REASONING***

a. The period is the time required for one complete cycle of the wave to pass. The period is also the time for two successive crests to pass the person.

b. The frequency is the reciprocal of the period, according to Equation 10.5.

c. The wavelength is the horizontal length of one cycle of the wave, or the horizontal distance between two successive crests.

d. The speed of the wave is equal to its frequency times its wavelength (see Equation 16.1).

e. The amplitude A of a wave is the maximum excursion of a water particle from the particle's undisturbed position.

SOLUTION

a. After the initial crest passes, 5 additional crests pass in a time of 50.0 s. The period T of the wave is

$$T = \frac{50.0\ \text{s}}{5} = \boxed{10.0\ \text{s}}$$

b. Since the frequency f and period T are related by $f = 1/T$ (Equation 10.5), we have

$$f = \frac{1}{T} = \frac{1}{10.0 \text{ s}} = \boxed{0.100 \text{ Hz}}$$

c. The horizontal distance between two successive crests is given as 32 m. This is also the wavelength λ of the wave, so

$$\lambda = \boxed{32 \text{ m}}$$

d. According to Equation 16.1, the speed v of the wave is

$$v = f\lambda = (0.100 \text{ Hz})(32 \text{ m}) = \boxed{3.2 \text{ m/s}}$$

e. There is no information given, either directly or indirectly, about the amplitude of the wave. Therefore,

$$\boxed{\text{it is not possible to determine the amplitude.}}$$

---

4. ***REASONING*** The speed of a Tsunamis is equal to the distance x it travels divided by the time t it takes for the wave to travel that distance. The frequency f of the wave is equal to its speed divided by the wavelength λ, $f = v/\lambda$ (Equation 16.1). The period T of the wave is related to its frequency by Equation 10.5, $T = 1/f$.

SOLUTION

a. The speed of the wave is (in m/s)

$$v = \frac{x}{t} = \frac{3700 \times 10^3 \text{ m}}{5.3 \text{ h}}\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = \boxed{190 \text{ m/s}}$$

b. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{190 \text{ m/s}}{750 \times 10^3 \text{ m}} = \boxed{2.5 \times 10^{-4} \text{ Hz}} \qquad (16.1)$$

c. The period of any wave is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{2.5 \times 10^{-4} \text{ Hz}} = \boxed{4.0 \times 10^3 \text{ s}} \qquad (10.5)$$

---

5. **SSM** ***REASONING*** When the end of the Slinky is moved up and down continuously, a transverse wave is produced. The distance between two adjacent crests on the wave, is, by definition, one wavelength. The wavelength λ is related to the speed and frequency of a periodic wave by $\lambda = v/f$ (Equation 16.1). In order to use Equation 16.1, we must first determine the frequency of the wave. The wave on the Slinky will have the same frequency as the simple harmonic motion of the hand. According to the data given in the problem statement, the frequency is $f = (2.00 \text{ cycles})/(1 \text{ s}) = 2.00 \text{ Hz}$.

SOLUTION Substituting the values for λ and f, we find that the distance between crests is

$$\lambda = \frac{v}{f} = \frac{0.50\text{ m/s}}{2.00\text{ Hz}} = \boxed{0.25\text{ m}}$$

---

6. ***REASONING*** The speed v of the wave is $v = f\lambda$ (Equation 16.1), where f is the frequency and λ is the wavelength of the wave. The frequency is $f = 1/T$ (Equation 10.5), where T is the period. The period is the time between successive crests. The wavelength is the distance between two successive crests.

SOLUTION Substituting Equation 10.5 for the frequency into Equation 16.1 for the speed, we obtain

$$v = f\lambda = \frac{\lambda}{T} \tag{1}$$

The wavelength is given as $\lambda = 4.0\text{ m}$. Since four crests pass by in 7.0 s and the period is the time between successive crests, the period is $T = (7.0\text{ s})/3$. Therefore, Equation (1) reveals that the speed of the wave is

$$v = \frac{\lambda}{T} = \frac{4.0\text{ m}}{(7.0\text{ s})/3} = \boxed{1.7\text{ m/s}}$$

---

7. ***REASONING*** The speed v of a wave is equal to its frequency f times its wavelength λ (Equation 16.1). The wavelength is the horizontal length of one cycle of the wave. From the left graph in the text it can be seen that this distance is 0.040 m. The frequency is the reciprocal of the period, according to Equation 10.5, and the period is the time required for one complete cycle of the wave to pass. From the right graph in the text, it can be seen that the period is 0.20 s, so the frequency is 1/(0.20 s).

SOLUTION Since the wavelength is λ = 0.040 m and the period is T = 0.20 s, the speed of the wave is

$$v = \lambda f = \lambda\left(\frac{1}{T}\right) = (0.040\text{ m})\left(\frac{1}{0.20\text{ s}}\right) = \boxed{0.20\text{ m/s}}$$

---

8. ***REASONING AND SOLUTION*** First find the speed of the record at a distance of 0.100 m from the center:

$$v = r\omega = (0.100\text{ m})(3.49\text{ rad/s}) = 0.349\text{ m/s} \tag{8.9}$$

The wavelength is, then,

$$\lambda = v/f = (0.349\text{ m/s})/(5.00 \times 10^3\text{ Hz}) = \boxed{6.98 \times 10^{-5}\text{ m}} \tag{16.1}$$

$$\lambda = (v_s - v_w)T_1 = (12.0 - v_w)(0.600) = 7.20 - 0.600\ v_w \qquad (1)$$

Similarly, for the skier traveling opposite the waves

$$\lambda = (v_s + v_w)T_2 = 6.00 + 0.500\ v_w \qquad (2)$$

a. Subtracting Equation (2) from Equation(1) and solving for v_w gives $v_w = \boxed{1.09 \text{ m/s}}$.

b. Substituting the value for v_w into Equation (1) gives $\lambda = \boxed{6.55 \text{ m}}$.

---

12. ***REASONING*** The length L of the string is one factor that affects the speed of a wave traveling on it, in so far as the speed v depends on the mass per unit length m/L according to $v = \sqrt{\dfrac{F}{m/L}}$ (Equation 16.2). The other factor affecting the speed is the tension F. The speed is not directly given here. However, the frequency f and the wavelength λ are given, and the speed is related to them according to $v = f\lambda$ (Equation 16.1). Substituting Equation 16.1 into Equation 16.2 will give an equation that can be solved for the length L.

SOLUTION Substituting Equation 16.1 into Equation 16.2 gives

$$v = f\lambda = \sqrt{\frac{F}{m/L}}$$

Solving for the length L, we find that

$$L = \frac{f^2\lambda^2 m}{F} = \frac{(260\text{ Hz})^2(0.60\text{ m})^2\left(5.0\times10^{-3}\text{ kg}\right)}{180\text{ N}} = \boxed{0.68\text{ m}}$$

---

13. SSM ***REASONING*** According to Equation 16.2, the linear density of the string is given by $(m/L) = F/v^2$, where the speed v of waves on the middle C string is given by Equation 16.1, $v = f\lambda = \left(\dfrac{1}{T}\right)\lambda$, where T is the period.

SOLUTION Combining Equations 16.2 and 16.1 and using the given data, we obtain

$$m/L = \frac{F}{v^2} = \frac{FT^2}{\lambda^2} = \frac{(944\text{ N})(3.82\times10^{-3}\text{ s})^2}{(1.26\text{ m})^2} = \boxed{8.68\times10^{-3}\text{ kg/m}}$$

---

14. ***REASONING*** The speed v of a transverse wave on a wire is given by $v = \sqrt{F/(m/L)}$ (Equation 16.2), where F is the tension and m/L is the mass per unit length (or linear density) of the wire. We are given that F and m are the same for the two wires, and that one is twice as long as the other. This information, along with knowledge of the wave speed on the shorter wire, will allow us to determine the speed of the wave on the longer wire.

SOLUTION The speeds on the longer and shorter wires are:

[Longer wire] $$v_{\text{longer}} = \sqrt{\frac{F}{m/L_{\text{longer}}}}$$

[Shorter wire] $$v_{\text{shorter}} = \sqrt{\frac{F}{m/L_{\text{shorter}}}}$$

Dividing the expression for v_{longer} by that for v_{shorter} gives

$$\frac{v_{\text{longer}}}{v_{\text{shorter}}} = \frac{\sqrt{\dfrac{F}{m/L_{\text{longer}}}}}{\sqrt{\dfrac{F}{m/L_{\text{shorter}}}}} = \sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}}$$

Since v_{shorter} = 240 m/s and $L_{\text{longer}} = 2L_{\text{shorter}}$, the speed of the wave on the longer wire is

$$v_{\text{longer}} = v_{\text{shorter}}\sqrt{\frac{L_{\text{longer}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{\frac{2L_{\text{shorter}}}{L_{\text{shorter}}}} = (240\text{ m/s})\sqrt{2} = \boxed{340\text{ m/s}}$$

---

15. ***REASONING*** The speed v of the transverse pulse on the wire is determined by the tension F in the wire and the mass per unit length m/L of the wire, according to $v = \sqrt{\dfrac{F}{m/L}}$ (Equation 16.2). The ball has a mass M. Since the wire supports the weight Mg of the ball and since the weight of the wire is negligible, it is only the ball's weight that determines the tension in the wire, $F = Mg$. Therefore, we can use Equation 16.2 with this value of the tension and solve it for the acceleration g due to gravity. The speed of the transverse pulse is not given, but we know that the pulse travels the length L of the wire in a time t and that the speed is $v = L/t$.

SOLUTION Substituting the tension $F = Mg$ and the speed $v = L/t$ into Equation 16.2 for the speed of the pulse on the string gives

$$v = \sqrt{\frac{F}{m/L}} \quad \text{or} \quad \frac{L}{t} = \sqrt{\frac{Mg}{m/L}}$$

Solving for the acceleration g due to gravity, we obtain

$$g = \frac{\left(\dfrac{L}{t}\right)^2 (m/L)}{M} = \frac{\left(\dfrac{0.95\text{ m}}{0.016\text{ s}}\right)^2 \left(1.2\times10^{-4}\text{ kg/m}\right)}{0.055\text{ kg}} = \boxed{7.7\text{ m/s}^2}$$

---

16. ***REASONING*** Each pulse travels a distance that is given by vt, where v is the wave speed and t is the travel time up to the point when they pass each other. The sum of the distances

32. ***REASONING*** The period T and the frequency f of the sound wave are related according to $f = 1/T$ (Equation 10.5). Therefore, if we can obtain the frequency, we can determine the frequency. Since the wavelength λ and the speed of sound v in seawater are given, we can obtain the frequency f from $v = f\lambda$ (Equation 16.1).

SOLUTION According to $f = 1/T$ (Equation 10.5), the period of the sound wave is

$$T = \frac{1}{f} \tag{1}$$

Solving $v = f\lambda$ (Equation 16.1) for the frequency gives

$$f = \frac{v}{\lambda} \tag{2}$$

Substituting Equation (2) into Equation (1), we find that

$$T = \frac{1}{f} = \frac{1}{v/\lambda} = \frac{\lambda}{v} = \frac{0.015\text{ m}}{1522\text{ m/s}} = \boxed{9.9 \times 10^{-6}\text{ s}}$$

---

33. ***REASONING AND SOLUTION*** The speed of sound in an ideal gas is given by text Equation 16.5

$$v = \sqrt{\frac{\gamma kT}{m}}$$

where m is the mass of a single gas particle (atom or molecule). Solving for T gives

$$T = \frac{mv^2}{\gamma k} \tag{1}$$

The mass of a single helium atom is

$$\frac{4.003\text{ g/mol}}{6.022\times10^{23}\text{ mol}^{-1}}\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = 6.650\times10^{-27}\text{ kg}$$

The speed of sound in oxygen at 0 °C is 316 m/s (see Table 16.1). Since helium is a monatomic gas, $\gamma = 1.67$. Then, substituting into Equation (1) gives

$$T = \frac{\left(6.65\times10^{-27}\text{ kg}\right)\left(316\text{ m/s}\right)^2}{1.67\left(1.38\times10^{-23}\text{ J/K}\right)} = \boxed{28.8\text{ K}}$$

---

34. ***REASONING*** A rail can be approximated as a long slender bar, so the speed of sound in the rail is given by Equation 16.7. With this equation and the given data for Young's modulus and the density of steel, we can determine the speed of sound in the rail. Then, we will be able to compare this speed to the speed of sound in air at 20 °C, which is 343 m/s.

39. ***REASONING*** Under the assumptions given in the problem, both the bullet and the sound of the rifle discharge travel at constant speeds. Therefore, we can use $v=\frac{d}{t}$ (Equation 2.1, where v is speed, d is distance, and t is elapsed time) to determine the distance that the bullet travels, as well as the time it takes the rifle report to reach the observer. The speed of the bullet is given, and we know that the speed of sound in air is 343 m/s, because the air temperature is 20 °C (See Table 16.1).

SOLUTION Applying Equation 2.1, we can express the distance d_b traveled by the bullet as

$$d_b = v_b t \tag{1}$$

where v_b = 840 m/s and t is the time needed for the report of the rifle to reach the observer. The observer is a distance $d = 25$ m from the marksman, so we can use Equation 2.1 with the speed of sound v = 343 m/s to determine t:

$$t = \frac{d}{v} \tag{2}$$

Substituting Equation (2) into Equation (1), we find the distance traveled by the bullet before the observer hears the report:

$$d_b = v_b t = v_b\left(\frac{d}{v}\right) = \frac{v_b d}{v} = \frac{(840\text{ m/s})(25\text{ m})}{343\text{ m/s}} = \boxed{61\text{ m}}$$

---

40. ***REASONING*** Generally, the speed of sound in a liquid like water is greater than in a gas. And, in fact, according to Table 16.1, the speed of sound in water is greater than the speed of sound in air. Since the speed of sound in water is greater than in air, an underwater ultrasonic pulse returns to the ruler in a shorter time than a pulse in air. The ruler has been designed for use in air, not in water, so this quicker return time fools the ruler into believing that the object is much closer than it actually is. Therefore, the reading on the ruler is less than the actual distance.

SOLUTION Let x be the actual distance from the ruler to the object. The time it takes for the ultrasonic pulse to reach the object and return to the ruler, a distance of $2x$, is equal to the distance divided by the speed of sound in water v_{water}: $t = 2x/v_{\text{water}}$. The speed of sound in water is given by Equation 16.6 as $v_{\text{water}} = \sqrt{B_{\text{ad}}/\rho}$, where B_{ad} is the adiabatic bulk modulus and ρ is the density of water. Thus, the time it takes for the pulse to return is

$$t = \frac{2x}{v_{\text{water}}} = \frac{2x}{\sqrt{\dfrac{B_{\text{ad}}}{\rho}}} = \frac{2(25.0\text{ m})}{\sqrt{\dfrac{2.37\times10^9\text{ Pa}}{1025\text{ kg/m}^3}}} = 3.29\times10^{-2}\text{ s}$$

The ruler measures this value for the time and computes the distance to the object by using the speed of sound in air, 343 m/s. The distance x_{ruler} displayed by the ruler is equal to the speed of sound in air multiplied by the time $\frac{1}{2}t$ it takes for the pulse to go from the ruler to the object:

$$x_{\text{ruler}} = v_{\text{air}}\left(\tfrac{1}{2}t\right) = (343\text{ m/s})\left(\tfrac{1}{2}\right)\left(3.29\times10^{-2}\text{ s}\right) = \boxed{5.64\text{ m}}$$

Thus, the ruler displays a distance of x_{ruler} = 5.64 m. As expected, the reading on the ruler's display is less than the actual distance of 25.0 m.

---

41. ***REASONING AND SOLUTION***

a. In order to determine the order of arrival of the three waves, we need to know the speeds of each wave. The speeds for air, water and the metal are

$$v_a = 343\text{ m/s},\ v_w = 1482\text{ m/s},\ v_m = 5040\text{ m/s}$$

The order of arrival is $\boxed{\text{metal wave first, water wave second, air wave third}}$.

b. Calculate the length of time each wave takes to travel 125 m.

$$t_m = (125\text{ m})/(5040\text{ m/s}) = 0.025\text{ s}$$
$$t_w = (125\text{ m})/(1482\text{ m/s}) = 0.084\text{ s}$$
$$t_a = (125\text{ m})/(343\text{ m/s}) = 0.364\text{ s}$$

Therefore, the delay times are

$$\Delta t_{12} = t_w - t_m = 0.084\text{ s} - 0.025\text{ s} = \boxed{0.059\text{ s}}$$
$$\Delta t_{13} = t_a - t_m = 0.364\text{ s} - 0.025\text{ s} = \boxed{0.339\text{ s}}$$

---

42. ***REASONING*** Since the sound wave travels twice as far in neon as in krypton in the same time, the speed of sound in neon must be twice that in krypton:

$$v_{\text{neon}} = 2v_{\text{krypton}} \qquad (1)$$

Furthermore, the speed of sound in an ideal gas is given by $v = \sqrt{\dfrac{\gamma kT}{m}}$, according to Equation 16.5. In this expression γ is the ratio of the specific heat capacities at constant pressure and constant volume and is the same for either gas (see Section 15.6), k is Boltzmann's constant, T is the Kelvin temperature, and m is the mass of an atom. This expression for the speed can be used for both gases in Equation (1) and the result solved for the temperature of the neon.

SOLUTION Using Equation 16.5 in Equation (1), we have

$$\underbrace{\sqrt{\frac{\gamma k T_{\text{neon}}}{m_{\text{neon}}}}}_{\text{Speed in neon}} = 2\underbrace{\sqrt{\frac{\gamma k T_{\text{krypton}}}{m_{\text{krypton}}}}}_{\text{Speed in krypton}}$$

Squaring this result and solving for the temperature of the neon give

53. SSM ***REASONING AND SOLUTION*** Since the sound radiates uniformly in all directions, at a distance r from the source, the energy of the sound wave is distributed over the area of a sphere of radius r. Therefore, according to $I = \frac{P}{4\pi r^2}$ (Equation 16.9) with $r = 3.8$ m, the power radiated from the source is

$$P = 4\pi I r^2 = 4\pi(3.6\times10^{-2}\ \text{W/m}^2)(3.8\ \text{m})^2 = \boxed{6.5\ \text{W}}$$

---

54. ***REASONING***

a. The source emits sound uniformly in all directions, so the sound intensity I at any distance r is given by Equation 16.9 as $I = P/\left(4\pi r^2\right)$, where P is the sound power emitted by the source. Since patches 1 and 2 are at the same distance from the source of sound, the sound intensity at each location is the same, so $I_1 = I_2$. Patch 3 is farther from the sound source, so the intensity I_3 is smaller for points on that patch. Therefore, patches 1 and 2 have equal intensities, each of which is greater than the intensity at patch 3.

b. According to Equation 16.8, the sound intensity I is defined as the sound power P that passes perpendicularly through a surface divided by the area A of that surface, $I = P/A$. The area of the surface is, then, $A = P/I$. Since the same sound power passes through patches 1 and 2, and the intensity at each one is the same, their areas must also be the same, $A_1 = A_2$. The same sound power passes through patch 3, but the intensity at that surface is smaller than that at patches 1 and 2. Thus, the area A_3 of patch 3 is larger than that of surface 1 or 2. In summary, A_3 is the largest area, followed by A_1 and A_2, which are equal.

SOLUTION

a. The sound intensity at the inner spherical surface is given by Equation 16.9 as

$$I_A = \frac{P}{4\pi r_A^2} = \frac{2.3\ \text{W}}{4\pi\left(0.60\ \text{m}\right)^2} = 0.51\ \text{W/m}^2$$

This intensity is the same at all points on the inner surface, since all points are equidistant from the sound source. Therefore, the sound intensity at patches 1 and 2 are equal; $I_1 = I_2 = \boxed{0.51\ \text{W/m}^2}$.

The sound intensity at the outer spherical surface is

$$I_B = \frac{P}{4\pi r_B^2} = \frac{2.3\ \text{W}}{4\pi\left(0.80\ \text{m}\right)^2} = 0.29\ \text{W/m}^2$$

This intensity is the same at all points on outer surface. Therefore, the sound intensity at patch 3 is $I_3 = \boxed{0.29\ \text{W/m}^2}$.

$$\beta_2 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) = (10\text{ dB})\log\left(\frac{8.0\times10^{-4}\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2}\right) = \boxed{89\text{ dB}}$$

where we have used $I_0 = 1.0\times10^{-12}$ W/m^2 for the threshold of hearing.

---

64. ***REASONING*** The sound intensity level β in decibels (dB) is related to the sound intensity I according to $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$ (Equation 16.10), where I_0 is the intensity of the reference level. We will apply this expression to each of the given intensity levels.

SOLUTION The sound intensity level is changed from $\beta_1 = 23\text{ dB}$ to $\beta_2 = 61\text{ dB}$. Therefore, the amount of the change is

$$\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_1}{I_0}\right)$$

It is a property of logarithms (see Equation D-12 in Appendix D) that $\log A - \log B = \log\left(\frac{A}{B}\right)$. Therefore, our expression for the change in sound intensity level becomes

$$\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) = (10\text{ dB})\log\left(\frac{I_2/I_0}{I_1/I_0}\right) = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$$

$$(61\text{ dB}) - (23\text{ dB}) = 38\text{ dB} = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right) \qquad \text{or} \qquad \log\left(\frac{I_2}{I_1}\right) = 3.8$$

Thus, according to Equation D-8 in Appendix D, we find that the desired ratio is

$$\frac{I_2}{I_1} = 10^{3.8} = \boxed{6300}$$

---

65. **SSM** ***REASONING*** This is a situation in which the intensities I_{man} and I_{woman} (in watts per square meter) detected by the man and the woman are compared using the intensity level β, expressed in decibels. This comparison is based on Equation 16.10, which we rewrite as follows:

$$\beta = (10\text{ dB})\log\left(\frac{I_{\text{man}}}{I_{\text{woman}}}\right)$$

SOLUTION Using Equation 16.10, we have

SOLUTION We proceed to solve for v_s and substitute the data given in the problem statement. Rearrangement gives

$$\frac{v_s}{v} = \frac{f_s}{f_o} - 1$$

Solving for v_s and noting that $f_o / f_s = 0.86$ yields

$$v_s = v\left(\frac{f_s}{f_o} - 1\right) = (343 \text{ m/s})\left(\frac{1}{0.86} - 1\right) = \boxed{56 \text{ m/s}}$$

---

78. ***REASONING*** The dolphin is the source of the clicks, and emits them at a frequency f_s. The marine biologist measures a lower, Doppler-shifted click frequency f_o, because the dolphin is swimming directly away. The difference between the frequencies is the source frequency minus the observed frequency: $f_s - f_o$. We will use $f_o = f_s / \left(1 + \frac{v_s}{v}\right)$ (Equation 16.12), where v_s is the speed of the dolphin and v is the speed of sound in seawater, to determine the difference between the frequencies.

SOLUTION Solving $f_o = f_s / \left(1 + \frac{v_s}{v}\right)$ (Equation 16.12) for the unknown source frequency f_s, we obtain

$$f_s = f_o\left(1 + \frac{v_s}{v}\right)$$

Therefore, the difference between the source and observed frequencies is

$$f_s - f_o = f_o\left(1 + \frac{v_s}{v}\right) - f_o = f_o\left[\left(1 + \frac{v_s}{v}\right) - 1\right]$$

$$= f_o\left(\frac{v_s}{v}\right) = (2500 \text{ Hz})\left(\frac{8.0 \text{ m/s}}{1522 \text{ m/s}}\right) = \boxed{13 \text{ Hz}}$$

---

79. ***REASONING*** This problem deals with the Doppler effect in a situation where the source of the sound is moving and the observer is stationary. Thus, the observed frequency is given by Equation 16.11 when the car is approaching the observer and Equation 16.12 when the car is moving away from the observer. These equations relate the frequency f_o heard by the observer to the frequency f_s emitted by the source, the speed v_s of the source, and the speed v of sound. They can be used directly to calculate the desired ratio of the observed frequencies. We note that no information is given about the frequency emitted by the source. We will see, however, that none is needed, since f_s will be eliminated algebraically from the solution.

SOLUTION Equations 16.11 and 16.12 are

$$f_o^{\text{Approach}} = f_s\left(\frac{1}{1-v_s/v}\right) \quad (16.11) \qquad f_o^{\text{Recede}} = f_s\left(\frac{1}{1+v_s/v}\right) \quad (16.12)$$

The ratio is

$$\frac{f_o^{\text{Approach}}}{f_o^{\text{Recede}}} = \frac{f_s\left(\frac{1}{1-v_s/v}\right)}{f_s\left(\frac{1}{1+v_s/v}\right)} = \frac{1+v_s/v}{1-v_s/v} = \frac{1+\frac{9.00\text{ m/s}}{343\text{ m/s}}}{1-\frac{9.00\text{ m/s}}{343\text{ m/s}}} = \boxed{1.054}$$

As mentioned in the ***REASONING***, the unknown source frequency f_s has been eliminated algebraically from this calculation.

---

80. ***REASONING*** The observed frequency changes because of the Doppler effect. As you drive toward the parked car (a stationary source of sound), the Doppler effect is that given by Equation 16.13. As you drive away from the parked car, Equation 16.14 applies.

SOLUTION Equations 16.13 and 16.14 give the observed frequency f_o in each case:

$$\underbrace{f_{\text{o, toward}} = f_s\left(1+v_o/v\right)}_{\text{Driving toward parked car}} \qquad \text{and} \qquad \underbrace{f_{\text{o, away}} = f_s\left(1-v_o/v\right)}_{\text{Driving away from parked car}}$$

Subtracting the equation on the right from the one on the left gives the change in the observed frequency:

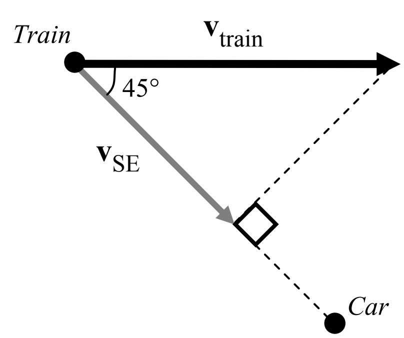
$$f_{\text{o, toward}} - f_{\text{o, away}} = 2 f_s v_o/v$$

Solving for the observer's speed (which is your speed), we obtain

$$v_o = \frac{v\left(f_{\text{o, toward}} - f_{\text{o, away}}\right)}{2 f_s} = \frac{(343\text{ m/s})(95\text{ Hz})}{2(960\text{ Hz})} = \boxed{17\text{ m/s}}$$

---

81. ***REASONING*** If the train were headed directly towards the car, the frequency f_o heard by the driver would be given by $f_o = f_s / \left(1-\frac{v_{\text{train}}}{v}\right)$ (Equation 16.11), where f_s is the frequency of the train's horn, v is the speed of sound in air, and v_{train} is the speed of the train. However, the driver is 20.0 m south of the crossing, and at the instant when the horn is sounded, the train is 20.0 m west of the crossing. The train would have to be headed directly southeast (45° south of east) in order to be moving directly towards the car at this instant. Therefore, to calculate the Doppler-shifted frequency of the horn blast, we must



Solutions for Home work Problems

Electromagnetic Waves : Chapter 24

7. ***REASONING AND SOLUTION*** Using Equation 16.1, we obtain

$$\lambda = \frac{c}{f} = \frac{2.9979\times 10^{8}\ \text{m/s}}{26.965\times 10^{6}\ \text{Hz}} = \boxed{11.118\ \text{m}}$$

---

8. ***REASONING*** According to Equation 16.1, the wavelength λ (in vacuum) is the speed of light c in a vacuum divided by the frequency f of the X-rays: $\lambda = \frac{c}{f}$.

SOLUTION Using Equation 16.1, we find that

$$\lambda = \frac{c}{f} = \frac{3.00\ \times\ 10^{8}\ \text{m/s}}{6.05\ \times\ 10^{18}\ \text{Hz}} = \boxed{4.96\ \times\ 10^{-11}\ \text{m}}$$

---

9. **SSM** ***REASONING*** The frequency f of the UHF wave is related to its wavelength by $c = f\lambda$ (Equation 16.1), where c is the speed of light in a vacuum and λ is the wavelength. The electric and magnetic fields are both zero at the same positions, which are separated by a distance d equal to half a wavelength (see Figure 24.3). Therefore, we can express the wavelength in terms of the distance between adjacent positions of zero field as

$$\lambda = 2d \tag{1}$$

SOLUTION Solving $c = f\lambda$ (Equation 16.1) for f yields

$$f = \frac{c}{\lambda} \tag{2}$$

Substituting Equation (1) into Equation (2), we obtain

$$f = \frac{c}{\lambda} = \frac{c}{2d} = \frac{3.00\times 10^{8}\ \text{m/s}}{2(0.34\ \text{m})} = \boxed{4.4\times 10^{8}\ \text{Hz}}$$

---

10. ***REASONING*** According to Equation 16.1, the wavelength λ (in vacuum) is the speed of light c in a vacuum divided by the frequency f of the radio waves: $\lambda = \frac{c}{f}$.

SOLUTION Using Equation 16.1, we find that the longest FM radio wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\ \times\ 10^{8}\ \text{m/s}}{88.0\ \times\ 10^{6}\ \text{Hz}} = \boxed{3.41\ \text{m}}$$

and back. The minimum, constant, angular speed of the mirror, then, can be found from $\omega = \frac{\Delta\theta}{\Delta t}$ (Equation 8.2). The time Δt it takes the light to travel from the rotating mirror to the fixed mirror and back is given by $c = \frac{2d}{\Delta t}$ (Equation 2.1), where $c = 3.00\times10^8$ m/s is the speed of light in a vacuum, and $d = 35$ km is the distance between the rotating mirror and the fixed mirror. Together, Equations 8.2 and 2.1 will allow us to determine the minimum angular speed ω of the rotating mirror. The angles between the rays of light shown in Figure 24.12 are exaggerated. In reality, the diameter of the rotating mirror is so much smaller than the distance d to the fixed mirror that these two rays may be considered to be parallel.

SOLUTION Solving $c = \frac{2d}{\Delta t}$ (Equation 2.1) for Δt yields

$$\Delta t = \frac{2d}{c} \tag{1}$$

Substituting Equation (1) into $\omega = \frac{\Delta\theta}{\Delta t}$ (Equation 8.2), we obtain

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta\theta}{\left(\frac{2d}{c}\right)} = \frac{c(\Delta\theta)}{2d} = \frac{(3.00\times10^8\ \text{m/s})(0.125\ \text{rev})}{2(35\times10^3\ \text{m})} = \boxed{540\ \text{rev/s}}$$

---

17. SSM ***REASONING*** We proceed by first finding the time t for sound waves to travel between the astronauts. Since this is the same time it takes for the electromagnetic waves to travel to earth, the distance between earth and the spaceship is $d_{\text{earth-ship}} = ct$.

SOLUTION The time it takes for sound waves to travel at 343 m/s through the air between the astronauts is

$$t = \frac{d_{\text{astronaut}}}{v_{\text{sound}}} = \frac{1.5\ \text{m}}{343\ \text{m/s}} = 4.4\times10^{-3}\ \text{s}$$

Therefore, the distance between the earth and the spaceship is

$$d_{\text{earth-ship}} = ct = (3.0\times10^8\ \text{m/s})(4.4\times10^{-3}\ \text{s}) = \boxed{1.3\times10^6\ \text{m}}$$

---

18. ***REASONING*** Let R denote the average rate at which the laptop downloads information, measured in bits per second (bps). This average rate is equal to the number N of bits downloaded in a time t divided by the time: $R = N/t$. Therefore, the number N of bits that the laptop downloads is given by

$$N = R\,t \tag{1}$$

We note that 1 Mbps (megabit per second) is equal to 10^6 bps. The time t is the time it takes the wireless signal to travel the distance $d = 8.1$ m between the router and the laptop. This time is determined by Equation 2.1 as

c. When the incident light is polarized along the *y* axis, the direction of polarization and the transmission axis are initially perpendicular to each other. The angle θ in Malus' law is the angle between the direction of polarization (along the *y* axis) and the transmission axis (measured relative to the *z* axis). It is related to the angle α according to $\theta = 90.0° - \alpha$. The average intensity of the transmitted light is, therefore,

$$\bar{S} = \bar{S}_0 \cos^2\theta = \left(7.0\ \text{W/m}^2\right)\cos^2\left(90.0° - 0°\right) = \boxed{0\ \text{W/m}^2}$$

$$\bar{S} = \bar{S}_0 \cos^2\theta = \left(7.0\ \text{W/m}^2\right)\cos^2\left(90.0° - 35°\right) = \boxed{2.3\ \text{W/m}^2}$$

The table below summarizes the results:

Incident Light	Average Intensity of Transmitted Light	
	$\alpha = 0°$	$\alpha = 35°$
(a) Unpolarized	3.5 W/m^2	3.5 W/m^2
(b) Polarized parallel to *z* axis	7.0 W/m^2	4.7 W/m^2
(c) Polarized parallel to *y* axis	0 W/m^2	2.3 W/m^2

---

45. ***REASONING*** Since the incident beam is unpolarized, the intensity of the light transmitted by the first sheet of polarizing material is one-half the intensity of the incident beam. The beams striking the second and third sheets of polarizing material are polarized, so the average intensity $\bar{S}$ of the light transmitted by each sheet is given by Malus' law, $\bar{S} = \bar{S}_0 \cos^2\theta$, where $\bar{S}_0$ is the average intensity of the light incident on each sheet.

SOLUTION The average intensity $\bar{S}_1$ of the light leaving the first sheet is one-half the intensity of the incident beam, so $\bar{S}_1 = \frac{1}{2}\left(1260.0\ \text{W/m}^2\right) = 630.0\ \text{W/m}^2$. The intensity $\bar{S}_2$ of the light leaving the second sheet of polarizing material is given by Malus' law, Equation 24.7, $\bar{S}_2 = \bar{S}_1 \cos^2\theta$, where θ is the angle between the polarization of the incident beam and the transmission axis of the second sheet:

$$\bar{S}_2 = \left(630.0\ \text{W/m}^2\right)\cos^2\left(55.0° - 19.0°\right) = 412\ \text{W/m}^2$$

The intensity $\bar{S}_3$ of the light leaving the third sheet of polarizing material is $\bar{S}_3 = \bar{S}_2 \cos^2\theta$, where θ is the angle between the polarization of the incident beam and the transmission axis of the third sheet:

$$\bar{S}_3 = \left(412\ \text{W/m}^2\right)\cos^2\left(100.0° - 55.0°\right) = \boxed{206\ \text{W/m}^2}$$

And the intensity of the light transmitted through the third analyzer is

$$\bar{S}_3 = \bar{S}_2 \cos^2 27° = \bar{S}_0 \cos^6 27°$$

If we generalize for the Nth analyzer, we deduce that

$$\bar{S}_N = \bar{S}_{N-1} \cos^2 27° = \bar{S}_0 \cos^{2N} 27°$$

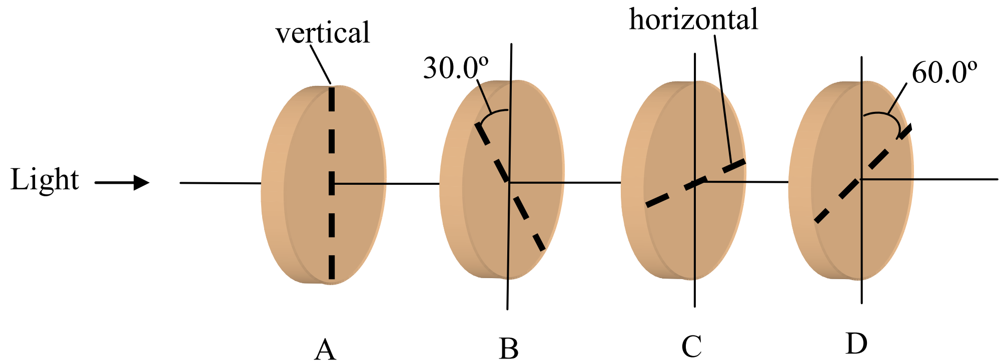
Since we want the light reaching the photocell to have an intensity that is reduced by a least a factor of one hundred relative to the first analyzer, we want $\bar{S}_N / \bar{S}_0 = 0.010$. Therefore, we need to find N such that $\cos^{2N} 27° = 0.010$. This expression can be solved for N.

SOLUTION Taking the common logarithm of both sides of the last expression gives

$$2N \log(\cos 27°) = \log 0.010 \quad \text{or} \quad N = \frac{\log 0.010}{2 \log(\cos 27°)} = \boxed{20}$$

---

48. ***REASONING*** No light intensity will pass through two adjacent polarizers that are in a crossed configuration, that is, whose transmission axes are oriented perpendicular to one another. With this in mind, let's remove the polarizers one by one (see the following drawing). When A is removed, no two of the remaining adjacent polarizers are crossed. When B is removed, A and C are left in a crossed configuration. When C is removed, B and D are left in a crossed configuration. When D is removed, no two of the remaining adjacent polarizers are crossed. Thus, when sheet B or C is removed, the intensity transmitted on the right is zero, and when sheet A or D is removed, the intensity transmitted on the right is greater than zero.



We can anticipate that the greater intensity is transmitted on the right when sheet D is removed. To begin with, we note that the polarization directions of the light striking B and C are the same, no matter whether A or D is removed, with the result that B and C absorb the same fraction of the intensity in either situation. When A is removed, however, D is a third sheet that absorbs light intensity. In contrast, when D is removed, A is present as a third sheet, but it absorbs none of the light intensity. This is because the transmission axis of A is vertical and matches the direction in which the incident light is polarized. We conclude, therefore, that the greater light intensity is transmitted when D is removed.

SOLUTION When light with an average intensity $\overline{S}_0$ is polarized at an angle θ with respect to the polarization axis of a polarizer, the average intensity $\overline{S}$ that is transmitted through the polarizer is given by Malus' law as $\overline{S} = \overline{S}_0 \cos^2\theta$ (Equation 24.7). The light that passes through is polarized in the direction of the transmission axis. In this problem, each of the polarizers, therefore, transmits a light intensity that is smaller than the incident light by a factor of $\cos^2\theta$. We use this insight now to determine the transmitted light intensity in the two situations that result when A is removed and when D is removed.

[A is removed; B, C, and D remain]

$$\overline{S} = \left(27\ \text{W/m}^2\right)\underbrace{\cos^2 30.0°}_{\text{due to B}}\underbrace{\cos^2 60.0°}_{\text{due to C}}\underbrace{\cos^2 30.0°}_{\text{due to D}} = \boxed{3.8\ \text{W/m}^2}$$

[B is removed; A, C, and D remain]

$$\overline{S} = \boxed{0\ \text{W/m}^2}$$

[C is removed; A, B, and D remain]

$$\overline{S} = \boxed{0\ \text{W/m}^2}$$

[D is removed; A B, and C remain]

$$\overline{S} = \left(27\ \text{W/m}^2\right)\underbrace{\cos^2 0.0°}_{\text{due to A}}\underbrace{\cos^2 30.0°}_{\text{due to B}}\underbrace{\cos^2 60.0°}_{\text{due to C}} = \boxed{5.1\ \text{W/m}^2}$$

---

49. ***REASONING*** The wavelength λ of a wave is related to its speed v and frequency f by $\lambda = v/f$ (Equation 16.1). Since blue light and orange light are electromagnetic waves, they travel through a vacuum at the speed of light c; thus, $v = c$.

SOLUTION
a. The wavelength of the blue light is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{6.34\times10^{14}\ \text{Hz}} = 4.73\times10^{-7}\ \text{m}$$

Since 1 nm = 10^{-9} m,

$$\lambda = \left(4.73\times10^{-7}\ \cancel{\text{m}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \cancel{\text{m}}}\right) = \boxed{473\ \text{nm}}$$

b. In a similar manner, we find that the wavelength of the orange light is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{4.95\times10^{14}\ \text{Hz}} = 6.06\times10^{-7}\ \text{m} = \boxed{606\ \text{nm}}$$

Solutions for Home work Problems

The Refraction of Light: Lenses and Optical Instruments : Chapter 26

Solving for v_B, we find that

$$v_B = v_A\left(\frac{n_A}{n_B}\right) = \left(1.25\times10^8\text{ m/s}\right)(1.33) = \boxed{1.66\times10^8\text{ m/s}}$$

---

4. ***REASONING*** The wavelength λ is related to the frequency f and speed v of the light in a material by Equation 16.1 ($\lambda = v/f$). The speed of the light in each material can be expressed using Equation 26.1 ($v = c/n$) and the refractive indices n given in Table 26.1. With these two equations, we can obtain the desired ratio.

SOLUTION Using Equations 16.1 and 26.1, we find

$$\lambda = \frac{v}{f} = \frac{c/n}{f} = \frac{c}{fn}$$

Using this result and recognizing that the frequency f and the speed c of light in a vacuum do not depend on the material, we obtain the ratio of the wavelengths as follows:

$$\frac{\lambda_{\text{alcohol}}}{\lambda_{\text{disulfide}}} = \frac{\left(\frac{c}{fn}\right)_{\text{alcohol}}}{\left(\frac{c}{fn}\right)_{\text{disulfide}}} = \frac{\frac{c}{f}\left(\frac{1}{n}\right)_{\text{alcohol}}}{\frac{c}{f}\left(\frac{1}{n}\right)_{\text{disulfide}}} = \frac{n_{\text{disulfide}}}{n_{\text{alcohol}}} = \frac{1.632}{1.362} = \boxed{1.198}$$

---

5. **SSM** ***REASONING*** The substance can be identified from Table 26.1 if its index of refraction is known. The index of refraction n is defined as the speed of light c in a vacuum divided by the speed of light v in the substance (Equation 26.1), both of which are known.

SOLUTION Using Equation 26.1, we find that

$$n = \frac{c}{v} = \frac{2.998\times10^8\text{ m/s}}{2.201\times10^8\text{ m/s}} = 1.362$$

Referring to Table 26.1, we see that the substance is $\boxed{\text{ethyl alcohol}}$.

---

6. ***REASONING*** We can identify the substance in Table 26.1 if we can determine its index of refraction. The index of refraction n is equal to the speed of light c in a vacuum divided by the speed of light v in the substance, or $n = c/v$. According to Equation 16.1, however, the speed of light is related to its wavelength λ and frequency f via $v = f\lambda$. Combining these two equations by eliminating the speed v yields $n = c/(f\lambda)$.

SOLUTION The index of refraction of the substance is

$$n = \frac{c}{f\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{\left(5.403 \times 10^{14} \text{ Hz}\right)\left(340.0 \times 10^{-9} \text{ m}\right)} = 1.632$$

An examination of Table 26.1 shows that the substance is $\boxed{\text{carbon disulfide}}$.

---

7. ***REASONING*** The refractive index n is defined by Equation 26.1 as $n = c/v$, where c is the speed of light in a vacuum and v is the speed of light in a material medium. The speed in a vacuum or in the liquid is the distance traveled divided by the time of travel. Thus, in the definition of the refractive index, we can express the speeds c and v in terms of the distances and the time. This will allow us to calculate the refractive index.

SOLUTION According to Equation 26.1, the refractive index is

$$n = \frac{c}{v}$$

Using d_{vacuum} and d_{liquid} to represent the distances traveled in a time t, we find the speeds to be

$$c = \frac{d_{\text{vacuum}}}{t} \quad \text{and} \quad v = \frac{d_{\text{liquid}}}{t}$$

Substituting these expressions into the definition of the refractive index shows that

$$n = \frac{c}{v} = \frac{d_{\text{vacuum}}/t}{d_{\text{liquid}}/t} = \frac{d_{\text{vacuum}}}{d_{\text{liquid}}} = \frac{6.20 \text{ km}}{3.40 \text{ km}} = \boxed{1.82}$$

---

8. ***REASONING*** Distance traveled is the speed times the travel time. Assuming that t is the time it takes for the light to travel through the two sheets, it would travel a distance of ct in a vacuum, where its speed is c. Thus, to find the desired distance, we need to determine the travel time t. This time is the sum of the travel times in each sheet. The travel time in each sheet is determined by the thickness of the sheet and the speed of the light in the material. The speed in the material is less than the speed in a vacuum and depends on the refractive index of the material.

SOLUTION In the ice of thickness d_{i}, the speed of light is v_{i}, and the travel time is $t_{\text{i}} = d_{\text{i}}/v_{\text{i}}$. Similarly, the travel time in the quartz sheet is $t_{\text{q}} = d_{\text{q}}/v_{\text{q}}$. Therefore, the desired distance ct is

$$ct = c\left(t_{\text{i}} + t_{\text{q}}\right) = c\left(\frac{d_{\text{i}}}{v_{\text{i}}} + \frac{d_{\text{q}}}{v_{\text{q}}}\right) = d_{\text{i}}\frac{c}{v_{\text{i}}} + d_{\text{q}}\frac{c}{v_{\text{q}}}$$

Since Equation 26.1 gives the refractive index as $n = c/v$ and since Table 26.1 gives the indices of refraction for ice and quartz as $n_{\text{i}} = 1.309$ and $n_{\text{q}} = 1.544$, the result just obtained can be written as follows:

$n_1 = 1.45$ and $\theta_1 = 64.0°$, whereas $\theta_2 = 53.0°$. We can use these data in Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Equation 26.2) to determine the unknown index of refraction n_2.

SOLUTION Using Snell's law we find that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{or} \quad n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{1.45 \sin 64.0°}{\sin 53.0°} = \boxed{1.63} \qquad (26.2)$$

---

11. SSM ***REASONING*** The angle of refraction θ_2 is related to the angle of incidence θ_1 by Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Equation 26.2), where n_1 and n_2 are, respectively the indices of refraction of the incident and refracting media. For each case (ice and water), the variables θ_1, n_1 and n_2, are known, so the angles of refraction can be determined.

SOLUTION The ray of light impinges from air (n_1 = 1.000) onto either the ice or water at an angle of incidence of θ_1 = 60.0°. Using n_2 = 1.309 for ice and n_2 = 1.333 for water, we find that the angles of refraction are

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

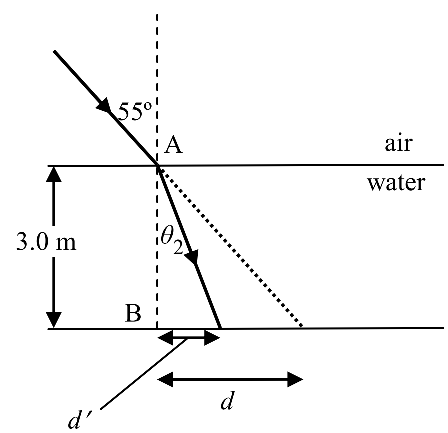
Ice $$\theta_{2,\text{ ice}} = \sin^{-1}\left[\frac{(1.000)\sin 60.0°}{1.309}\right] = 41.4°$$

Water $$\theta_{2,\text{ water}} = \sin^{-1}\left[\frac{(1.000)\sin 60.0°}{1.333}\right] = 40.5°$$

The difference in the angles of refraction is $\theta_{2,\text{ ice}} - \theta_{2,\text{ water}} = 41.4° - 40.5° = \boxed{0.9°}$

---

12. ***REASONING*** If refraction did not occur, the light would travel straight ahead, as indicated by the dotted path in the drawing at the right. It would then strike the lake-bottom at a distance *d* from point B. Because of refraction, however, the light ray is bent toward the dashed normal to the air-water surface, the angle of refraction being θ_2. With refraction, the light strikes the lake-bottom at a distance *d′* from point B. We will use trigonometry to find the distances *d* and *d′*. To find the angle of refraction θ_2, we will use Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Equation 26.2).



14. ***REASONING AND SOLUTION*** Using Equation 26.3, we find

$$d = \left(\frac{n_1}{n_2}\right)d' = \left(\frac{1.546}{1.000}\right)2.5 \text{ cm} = \boxed{3.9 \text{ cm}}$$

---

15. **SSM** ***REASONING*** We begin by using Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$) to find the index of refraction of the material. Then we will use Equation 26.1, the definition of the index of refraction ($n = c/v$) to find the speed of light in the material.

SOLUTION From Snell's law, the index of refraction of the material is

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1.000)\sin 63.0°}{\sin 47.0°} = 1.22$$

Then, from Equation 26.1, we find that the speed of light v in the material is

$$v = \frac{c}{n_2} = \frac{3.00\times10^8 \text{ m/s}}{1.22} = \boxed{2.46\times10^8 \text{ m/s}}$$

---

16. ***REASONING*** When the light ray passes from *a* into *b*, it is bent toward the normal. According to the discussion in Section 26.2, this happens when the index of refraction of *b* is greater than that of *a*, or $n_b > n_a$. When the light passes from *b* into *c*, it is bent away from the normal. This means that the index of refraction of *c* is less than that of *b*, or $n_c < n_b$. The smaller the value of n_c, the greater is the angle of refraction. As can be seen from the drawing, the angle of refraction in material *c* is greater than the angle of incidence at the *a*-*b* interface. Applying Snell's law to the *a*-*b* and *b*-*c* interfaces gives $n_a \sin\theta_a = n_b \sin\theta_b = n_c \sin\theta_c$. Since θ_c is greater than θ_a, the equation $n_a \sin\theta_a = n_c \sin\theta_c$ shows that the index of refraction of *a* must be greater than that of *c*, $n_a > n_c$. Thus, the ordering of the indices of refraction, highest to lowest, is n_b, n_a, n_c.

SOLUTION The index of refraction for each medium can be evaluated from Snell's law, Equation 26.2:

a-b interface

$$n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.20)\sin 50.0°}{\sin 45.0°} = \boxed{1.30}$$

b-c interface

$$n_c = \frac{n_b \sin\theta_b}{\sin\theta_c} = \frac{(1.30)\sin 45.0°}{\sin 56.7°} = \boxed{1.10}$$

As expected, the ranking of the indices of refraction, highest to lowest, is

$$n_b = 1.30,\ n_a = 1.20,\ n_c = 1.10$$

---

of this equation. Therefore, the liquid with the largest index of refraction has the smallest value of $\sin\theta_c$ and, correspondingly, the smallest value of the critical angle.

SOLUTION In Table 26.1 the liquid with the largest index of refraction is carbon disulfide. Using Equation 26.4 and taking the refractive index for carbon disulfide from Table 26.1, we obtain

$$\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{liquid}}} = \frac{1.000}{1.632} \qquad \text{or} \qquad \theta_c = \sin^{-1}\left(\frac{1.000}{1.632}\right) = \boxed{37.79°}$$

---

27. **SSM** ***REASONING*** The light ray traveling in the oil can only penetrate into the water if it does not undergo total internal reflection at the boundary between the oil and the water. Total internal reflection will occur if the angle of incidence $\theta = 71.4°$ is greater than the critical angle θ_c for these two media. The critical angle is found from

$$\sin\theta_c = \frac{n_2}{n_1} \qquad (26.4)$$

where $n_2 = 1.333$ is the index of refraction of water (see Table 26.1), and $n_1 = 1.47$ is the index of refraction of the oil.

SOLUTION Solving Equation 26.4 for θ_c, we obtain

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.333}{1.47}\right) = \boxed{65.1°}$$

Comparing this result to $\theta = 71.4°$, we see that the angle of incidence is greater than the critical angle ($\theta > \theta_c$). Therefore, the ray of light will not enter the water; it will instead undergo total internal reflection within the oil.

---

28. ***REASONING AND SOLUTION*** Only the light which has an angle of incidence less than or equal θ_c can escape. This light leaves the source in a cone whose apex angle is $2\theta_c$. The radius of this cone at the surface of the water (n = 1.333, see Table 26.1) is $R = d \tan\theta_c$. Now

$$\theta_c = \sin^{-1}\left(\frac{1.000}{1.333}\right) = 48.6°$$

so

$$R = (2.2 \text{ m}) \tan 48.6° = \boxed{2.5 \text{ m}}$$

---

29. ***REASONING AND SOLUTION***

a. The index of refraction n_2 of the liquid must match that of the glass, or $n_2 = \boxed{1.50}$.

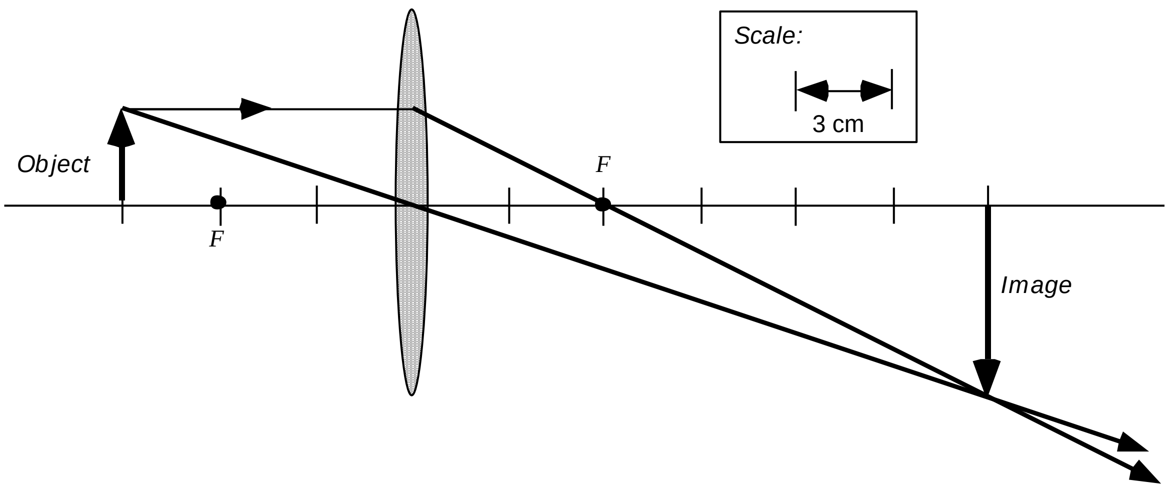
b. All colors except violet are to emerge from the slanted face. Therefore, we will use $n_1 = 1.531$, the index of refraction of blue light in crown glass (see Table 26.2), in Equation (1):

$$n_2 = (1.531)\sin 45.00^\circ = \boxed{1.083}$$

---

49. SSM ***REASONING*** The ray diagram is constructed by drawing the paths of two rays from a point on the object. For convenience, we choose the top of the object. The ray that is parallel to the principal axis will be refracted by the lens and pass through the focal point on the right side. The ray that passes through the center of the lens passes through undeflected. The image is formed at the intersection of these two rays on the right side of the lens.

SOLUTION The following ray diagram (to scale) shows that $\boxed{d_i = 18 \text{ cm}}$ and reveals a real, inverted, and enlarged image.



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50. ***REASONING***

a. Given the focal length ($f = -0.300$ m) of the lens and the image distance d_i, we will employ the thin-lens equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ (Equation 26.6) to determine how far from the van the person is actually standing, which is the object distance d_o. The image and the person are both behind the van, so the image is virtual, and the image distance is negative: $d_i = -0.240$ m.

b. Once we have determined the object distance d_o, we will use the magnification equation $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ (Equation 26.7) to calculate the true height h_o of the person from the height $h_i = 0.34$ m of the image.

SOLUTION
a. Solving Equation 26.6 for d_o, we obtain

$$\frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i} \quad \text{or} \quad d_o=\frac{1}{\frac{1}{f}-\frac{1}{d_i}}=\frac{1}{\frac{1}{-0.300\text{ m}}-\frac{1}{(-0.240\text{ m})}}=\boxed{1.2\text{ m}}$$

b. Taking the reciprocal of both sides of $\frac{h_i}{h_o}=-\frac{d_i}{d_o}$ (Equation 26.7) and solving for h_o yields

$$\frac{h_o}{h_i}=-\frac{d_o}{d_i} \quad \text{or} \quad h_o=-h_i\left(\frac{d_o}{d_i}\right)=-(0.34\text{ m})\left(\frac{1.2\text{ m}}{-0.240\text{ m}}\right)=\boxed{1.7\text{ m}}$$

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51. ***REASONING*** We can use the magnification equation (Equation 26.7) to determine the image height h_i. This equation is

$$\frac{h_i}{h_o}=-\frac{d_i}{d_o} \quad \text{or} \quad h_i=h_o\left(-\frac{d_i}{d_o}\right) \tag{26.7}$$

We are given the object height h_o and the object distance d_o. Thus, we need to begin by finding the image distance d_i, for which we use the thin-lens equation (Equation 26.6):

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \text{or} \quad \frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{d_o-f}{fd_o} \quad \text{or} \quad d_i=\frac{fd_o}{d_o-f} \tag{26.6}$$

Substituting this result into Equation 26.7 gives

$$h_i=h_o\left(-\frac{d_i}{d_o}\right)=h_o\left(-\frac{1}{d_o}\right)\left(\frac{fd_o}{d_o-f}\right)=h_o\left(\frac{f}{f-d_o}\right) \tag{1}$$

SOLUTION
a. Using Equation (1), we find that the image height for the 35.0-mm lens is

$$h_i=h_o\left(\frac{f}{f-d_o}\right)=(1.60\text{ m})\left[\frac{35.0\times10^{-3}\text{ m}}{(35.0\times10^{-3}\text{ m})-9.00\text{ m}}\right]=\boxed{-0.00625\text{ m}}$$

b. Using Equation (1), we find that the image height for the 150.0-mm lens is

$$h_i=h_o\left(\frac{f}{f-d_o}\right)=(1.60\text{ m})\left[\frac{150.0\times10^{-3}\text{ m}}{(150.0\times10^{-3}\text{ m})-9.00\text{ m}}\right]=\boxed{-0.0271\text{ m}}$$

Both heights are negative because the images are inverted with respect to the object.

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52. ***REASONING*** The focal length f can be found with the aid of the thin-lens equation $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$ (Equation 26.6), where d_o is the object distance and d_i is the image distance.

SOLUTION
a. The diverging lens produces a $\boxed{\text{virtual image}}$.

b. We know that $d_o = 13.0\text{ cm}$ and $d_i = -5.0\text{ cm}$, where the image distance is negative because the image is virtual. Using the thin lens equation, we find

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \text{or} \quad \frac{d_i+d_o}{d_o d_i}=\frac{1}{f} \quad \text{or} \quad f=\frac{d_o d_i}{d_i+d_o}=\frac{(13.0\text{ cm})(-5.0\text{ cm})}{(-5.0\text{ cm})+(13.0\text{ cm})}=\boxed{-8.1}$$

53. ***REASONING*** The distance from the lens to the screen, the image distance, can be obtained directly from the thin-lens equation, Equation 26.6, since the object distance and focal length are known. The width and height of the image on the screen can be determined by using Equation 26.7, the magnification equation.

SOLUTION
a. The distance d_i to the screen is

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{105.00\text{ mm}}-\frac{1}{108.00\text{ mm}}=2.646\times10^{-4}\text{ mm}^{-1}$$

so that $d_i = 3.78\times10^{3}\text{ mm} = \boxed{3.78\text{ m}}$.

b. According to the magnification equation, the width and height of the image on the screen are

Width
$$h_i = h_o\left(-\frac{d_i}{d_o}\right)=(24.0\text{ mm})\left(-\frac{3.78\times10^{3}\text{ mm}}{108\text{ mm}}\right)=-8.40\times10^{2}\text{ mm}$$

The width is $\boxed{8.40\times10^{2}\text{ mm}}$.

Height
$$h_i = h_o\left(-\frac{d_i}{d_o}\right)=(36.0\text{ mm})\left(-\frac{3.78\times10^{3}\text{ mm}}{108\text{ mm}}\right)=-1.26\times10^{3}\text{ mm}$$

The height is $\boxed{1.26\times10^{3}\text{ mm}}$.

Solutions for Home work Problems

The Nature of the Atom : Chapter 30

$$\frac{1}{\lambda}=\frac{2\pi^2 mk^2e^4}{h^3c}\,(Z^2)\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \quad \text{with} \quad n_i, n_f = 1,\ 2,\ 3,\ldots \quad \text{and} \quad n_i > n_f$$

where $2\pi^2 mk^2e^4/(h^3c)=1.097\times10^7\ \text{m}^{-1}$ and $Z = 1$ for hydrogen. Once the wavelength for the particular transition in question is determined, Equation 29.2 ($E = hf = hc/\lambda$) can be used to find the energy of the emitted photon.

SOLUTION In the Paschen series, $n_f = 3$. Using the above expression with $Z = 1$, $n_i = 7$ and $n_f = 3$, we find that

$$\frac{1}{\lambda}=\left(1.097\times10^7\ \text{m}^{-1}\right)\left(1^2\right)\left(\frac{1}{3^2}-\frac{1}{7^2}\right) \qquad \text{or} \qquad \lambda = 1.005\times10^{-6}\ \text{m}$$

The photon energy is

$$E=\frac{hc}{\lambda}=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{1.005\times10^{-6}\ \text{m}}=\boxed{1.98\times10^{-19}\ \text{J}}$$

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12. ***REASONING*** The ionization energy for a given state is the energy needed to remove the electron completely from the atom. The removed electron has no kinetic energy and no electric potential energy, so its total energy is zero.

The ionization energy for a given excited state is less than the ionization energy for the ground state. In the excited state the electron already has part of the energy necessary to achieve ionization, so less energy is required to ionize the atom from the excited state than from the ground state.

SOLUTION

a. The energy of the n^{th} state in the hydrogen atom is given by Equation 30.13 as $E_n = -(13.6\ \text{eV})Z^2/n^2$. When $n = \infty$, $E_\infty = 0\ \text{J}$, and when $n = 4$, $E_4 = -(13.6\ \text{eV})(1)^2/4^2 = -0.850\ \text{eV}$. The difference in energies between these two states is the ionization energy:

$$\text{Ionization energy} = E_\infty - E_4 = \boxed{0.850\ \text{eV}}$$

b. In the same manner, it can be shown that the ionization energy for the $n = 1$ state is 13.6 eV. The ratio of the ionization energies is

$$\frac{0.850\ \text{eV}}{13.6\ \text{eV}} = \boxed{0.0625}$$

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13. ***REASONING*** Since the atom emits two photons as it returns to the ground state, one is emitted when the electron falls from n = 3 to n = 2, and the other is emitted when it subsequently drops from n = 2 to n = 1. The wavelengths of the photons emitted during these transitions are given by Equation 30.14 with the appropriate values for the initial and final numbers, n_i and n_f.

SOLUTION The wavelengths of the photons are

n = 3 to n = 2

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\ \text{m}^{-1}\right)(1)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=1.524\times10^{6}\ \text{m}^{-1} \qquad (30.14)$$

$$\lambda=\boxed{6.56\times10^{-7}\ \text{m}}$$

n = 2 to n = 1

$$\frac{1}{\lambda}=\left(1.097\times10^{7}\ \text{m}^{-1}\right)(1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=8.228\times10^{6}\ \text{m}^{-1} \qquad (30.14)$$

$$\lambda=\boxed{1.22\times10^{-7}\ \text{m}}$$

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14. ***REASONING*** The energy levels in a hydrogen atom are given by $E_n=-(13.6\ \text{eV})\dfrac{Z^2}{n^2}$ (Equation 30.13), where $Z=1$ is the number of protons in the hydrogen nucleus and $n=1,\ 2,\ 3,\ \ldots$ .To use Equation 30.13, we need a value for the quantum number n. We can obtain this value from the radius r_n , since $r_n=\left(5.29\times10^{-11}\ \text{m}\right)\dfrac{n^2}{Z}$ (Equation 30.10).

SOLUTION Solving Equation 30.10 for n^2, we have

$$r_n=\left(5.29\times10^{-11}\ \text{m}\right)\frac{n^2}{Z} \qquad \text{or} \qquad n^2=\frac{r_nZ}{5.29\times10^{-11}\ \text{m}} \qquad (1)$$

Substituting Equation (1) into Equation 30.13, we find

$$E_n=-(13.6\ \text{eV})\frac{Z^2}{n^2}=-(13.6\ \text{eV})\frac{Z^2}{r_nZ/\left(5.29\times10^{-11}\ \text{m}\right)}$$

$$=-(13.6\ \text{eV})\frac{Z\left(5.29\times10^{-11}\ \text{m}\right)}{r_n}=-(13.6\ \text{eV})\frac{(1)\left(5.29\times10^{-11}\ \text{m}\right)}{\left(4.761\times10^{-10}\ \text{m}\right)}=\boxed{-1.51\ \text{eV}}$$

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15. ***REASONING*** The Bohr expression as it applies to any one-electron species of atomic number Z, is given by Equation 30.13: $E_n=-(13.6\ \text{eV})(Z^2/n^2)$. For certain values of the quantum number n, this expression predicts equal electron energies for singly ionized

SOLUTION Of the five states listed in the table, three are not possible. The ones that are not possible, and the reasons they are not possible, are:

State	Reason
(a)	The quantum number ℓ must be less than n
(c)	The quantum number m_ℓ must be less than or equal to ℓ .
(d)	The quantum number ℓ cannot be negative.

---

25. ***REASONING*** The orbital quantum number ℓ has values of 0, 1, 2, ..., (n – 1), according to the discussion in Section 30.5. Since $\ell = 5$, we can conclude, therefore, that $n \geq 6$. This knowledge about the principal quantum number n can be used with Equation 30.13, $E_n = -(13.6\text{ eV})Z^2/n^2$, to determine the smallest value for the total energy E_n.

SOLUTION The smallest value of E_n (i.e., the most negative) occurs when $n = 6$. Thus, using $Z = 1$ for hydrogen, we find

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} = -(13.6\text{ eV})\frac{1^2}{6^2} = \boxed{-0.378\text{ eV}}$$

---

26. ***REASONING*** We can determine the orbital quantum number ℓ from the maximum allowed magnitude of the magnetic quantum number m_ℓ (see Section 30.5). For a given value of ℓ the allowed values for m_ℓ are

$$-\ell, \ldots, -2, -1, 0, +1, +2, \ldots, +\ell$$

Once the value for ℓ is identified, we can determine what values for the principal quantum number n are possible (see Section 30.5). For a given value of n, the permissible values for ℓ are

$$\ell = 0, 1, 2, 3, \ldots, (n-1)$$

SOLUTION Since the maximum magnitude for the magnetic quantum number is $|m_\ell| = 4$, we conclude that $\boxed{\ell = 4}$. Furthermore, we know that the value for ℓ must be less than or equal to $n - 1$. Therefore, the smallest possible value of the principal quantum number is $\boxed{n = 5}$.

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27. **SSM** ***REASONING*** The maximum value for the magnetic quantum number is $m_\ell = \ell$; thus, in state A, $\ell = 2$, while in state B, $\ell = 1$. According to the quantum mechanical theory

two are related. Thus, we will be able to evaluate the wavelength of the X-ray photon from a knowledge of the electron's speed.

SOLUTION As discussed in Section 16.2, the wavelength λ of a wave is related to its frequency f and speed v by $\lambda = v/f$ (Equation 16.1). In a vacuum, an electromagnetic wave travels at the speed c of light. Substituting $v = c$ into $\lambda = v/f$ gives $\lambda = c/f$. An X-ray photon is an electromagnetic wave that is a discrete packet of energy. The photon's frequency f is related to its energy E by $f = E/h$ (Equation 29.2), where h is Planck's constant. Substituting this expression for f into $\lambda = c/f$ gives

$$\lambda = \frac{c}{f} = \frac{c}{\left(\frac{E}{h}\right)} = \frac{ch}{E} \tag{1}$$

The energy needed to produce an X-ray photon comes from the kinetic energy of an electron striking the target. If the speed of the electron is much less than the speed of light in a vacuum, its kinetic energy KE given by Equation 6.2 as $\text{KE} = \frac{1}{2}mv^2$, where m and v are the mass and speed of the electron. We are given that the electron decelerates to one-quarter of its original speed, so that its loss of kinetic energy is

$$\text{Loss of kinetic energy} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}m\left(\tfrac{1}{4}v\right)^2 = \tfrac{15}{32}mv^2$$

The kinetic energy lost by the decelerating electron goes into creating the X-ray photon, so that $E = \frac{15}{32}mv^2$. Substituting this expression for E into Equation (1) gives

$$\lambda = \frac{ch}{E} = \frac{ch}{\frac{15}{32}mv^2} = \frac{\left(3.00\times10^{8}\text{ m/s}\right)\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)}{\frac{15}{32}\left(9.11\times10^{-31}\text{ kg}\right)\left(6.00\times10^{7}\text{ m/s}\right)^2} = \boxed{1.29\times10^{-10}\text{ m}}$$

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45. **SSM** ***REASONING*** The number of photons emitted by the laser will be equal to the total energy carried in the beam divided by the energy per photon.

SOLUTION The total energy carried in the beam is, from the definition of power,

$$E_{\text{total}} = Pt = (1.5\text{ W})(0.050\text{ s}) = 0.075\text{ J}$$

The energy of a single photon is given by Equations 29.2 and 16.1 as

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{514\times10^{-9}\text{ m}} = 3.87\times10^{-19}\text{ J}$$

where we have used the fact that $514\text{ nm} = 514\times10^{-9}\text{ m}$. Therefore, the number of photons emitted by the laser is

$$\frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{0.075\text{ J}}{3.87\times10^{-19}\text{ J/photon}} = \boxed{1.9\times10^{17}\text{ photons}}$$

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46. ***REASONING*** The energy E delivered a single laser photon is $E = hf$ (Equation 29.2), where $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$ is Planck's constant and f is the frequency. The frequency is related to the wavelength λ according to $f\lambda = c$ (Equation 16.1), so that $f = c/\lambda$, where $c = 3.00\times10^{8}\text{ m/s}$ is the speed of light in a vacuum. Substituting this expression for the frequency into Equation 29.2 gives the following equation for the energy of a single photon:

$$E = hf = \frac{hc}{\lambda} \tag{1}$$

The total energy of N photons is NE.

SOLUTION Using Equation (1), we set the total energy delivered by N photons from the carbon dioxide laser equal to the energy delivered by a single photon from the excimer laser:

$$\underbrace{N\frac{hc}{\lambda_{\text{carbon dioxide}}}}_{\substack{\text{Total energy of } N \text{ photons} \\ \text{from carbon dioxide laser}}} = \underbrace{\frac{hc}{\lambda_{\text{excimer}}}}_{\substack{\text{Energy of 1 photon} \\ \text{from carbon dioxide laser}}}$$

Solving for N, we find that

$$N = \frac{\lambda_{\text{carbon dioxide}}}{\lambda_{\text{excimer}}} = \frac{1.06\times10^{-5}\text{ m}}{193\times10^{-9}\text{ m}} = 54.9$$

Since only a whole number of photons is possible, the minimum number of photons required of the carbon dioxide laser is $\boxed{55}$.

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47. ***REASONING*** The total energy E_{tot} of a single pulse is equal to the number N of photons in the pulse multiplied by the energy E of each photon:

$$E_{\text{tot}} = NE \tag{1}$$

The energy E of each photon is given by $E = hf$ (Equation 29.2), where $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$ is Planck's constant and f is the frequency of the photon. We will use $f = \dfrac{c}{\lambda}$ (Equation 16.1) to determine the frequency f of the photons from their wavelength λ and the speed c of light in a vacuum. To find the total energy E_{tot} of each pulse, we will make use of the fact that the average power P_{avg} of the laser is equal to the total energy of a single pulse divided by the duration Δt of the pulse:

$$P_{\text{avg}} = \frac{E_{\text{tot}}}{\Delta t} \tag{6.10b}$$

SOLUTION Solving Equation (1) for *N*, we obtain

$$N = \frac{E_{\text{tot}}}{E} \tag{2}$$

Solving Equation (6.10b) for E_{tot} yields $E_{\text{tot}} = P_{\text{avg}}\Delta t$. Substituting this result and $E = hf$ (Equation 29.2) into Equation (2), we find that

$$N = \frac{E_{\text{tot}}}{E} = \frac{P_{\text{avg}}\Delta t}{hf} \tag{3}$$

Substituting $f = \dfrac{c}{\lambda}$ (Equation 16.1) into Equation (3), we find that the number of photons in each pulse is:

$$N = \frac{P_{\text{avg}}\Delta t}{hf} = \frac{P_{\text{avg}}\Delta t}{h\left(\dfrac{c}{\lambda}\right)} = \frac{P_{\text{avg}}\Delta t\lambda}{hc} = \frac{\left(5.00\times10^{-3}\text{ W}\right)\left(25.0\times10^{-3}\text{ s}\right)\left(633\times10^{-9}\text{ m}\right)}{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$$

$$= \boxed{3.98\times10^{14}}$$

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48. ***REASONING*** The external source of energy must "pump" the electrons from the ground state E_0 to the metastable state E_2. The population inversion occurs between the metastable state E_2 and the one below it, E_1. The lasing action occurs between the two states that have the population inversion, the E_2 and E_1 states.

SOLUTION

a. From the drawing, we see that the energy required to raise an electron from the E_0 state to the E_2 state is $\boxed{0.289\text{ eV}}$.

b. The lasing action occurs between the E_2 and E_1 states, and so the energy *E* of the emitted photon is the difference between them; $E = E_2 - E_1$. According to Equations 29.2 and 16.1, the wavelength λ of the photon is related to its energy via $\lambda = hc/E$, so that

$$\lambda = \frac{hc}{E_2 - E_1} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.289\text{ eV} - 0.165\text{ eV}\right)\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{1.00\times10^{-5}\text{ m}}$$